Match or Overmatch: How Academic Standard and School Prestige Affect School Choices

Xuejuan Su*

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Abstract

This paper analyzes the possibility and impact of over-match between students and schools. Schools differ in curriculum design, where high academic standard implies high qualification threshold and fast progress rate. Without school prestige, a student chooses school optimally where curriculum matches best with his qualification. With school prestige, a student may choose to overmatch, where the curriculum is too challenging but the loss in education achievement is more than offset by the gain in school prestige. Among students with identical qualifications, the less hardworking ones are more likely to over-match. This self-selection problem calls for a caveat in interpreting empirical relationship from observational data.

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*Author address: Department of Economics, Finance and Legal Studies, the University of Alabama, Box 870224, Tuscaloosa, AL 35487-0224. Email address: xsu@cba.ua.edu.

1 Introduction

A recent study by Robert Sander (2005) has generated much heated debate on whether affirmative action actually hurts its intended beneficiaries or not. The argument seems fairly straightforward. Affirmative action gives preferential treatment to minority students, so that they are admitted into schools with lower qualifications than their majority counterparts. The overmatch between minority students and schools are actually detrimental to their education outcomes, in that imposing too challenging a program on less prepared students leads to inferior performance.

Conventional wisdom aside, - we all take it for granted that different curricula are needed to best suit students with different qualifications, that's why college students don't share a class with kindergartners - it is surprising how little attention has been devoted to the issue of optimal match between students and schools/curricula. Besides differing in school quality measured as education expenditure, teacher qualification and/or peer effect, an important yet often overlooked aspect of school differentiation is its curriculum design. A curriculum imposes certain qualification threshold on students, together with certain progress rate. One familiar example would be teaching undergraduate intermediate microeconomics. Even with the same school quality, for a given student with given qualification, it makes a huge difference whether the course merely hands out the formula $\frac{MU_1}{MU_2} = \frac{p_i}{p_2}$ for utility maximization, or it introduces Lagrange to derive the first order condition, or it advances to Slutsky decomposition. Obviously a well prepared student with solid knowledge in calculus may deepen his understanding of the subject best from Slutsky decomposition, yet a less prepared student with shaky ability in taking partial derivatives may learn best simply equating marginal rate of substitution to relative price, without all the mumble jumble Lagrange or Slutsky distractions.

In this sense, the current paper develops a theoretical model where different curricula are represented by different education technologies. Both academic standard and progress rate are modeled as intrinsic to a specific technology instead of being measured by the education outcome. Hence heterogeneous students find different curricula best suited to their own qualifications. It is analogous to the comparison between absolute advantage and comparative advantage. For a given curriculum, students with higher qualifications always outperform students with lower qualifications, and this is their absolute advantage. Yet among different curricula, students with higher qualifications are better suited by more challenging programs, while students with lower qualifications are better suited by less challenging programs, and this is their comparative advantage. Thus the optimal match between students and schools is the match where students enjoy the highest learning efficiency with their qualifications under a suitable curriculum.

Besides curriculum, when schools also differ in their prestige, student school choice may deviate from the optimal qualification/curriculum match. If school prestige depends positively on the "eliteness" of the program, or equivalently how challenging the curriculum is, then students may be tempted to overmatch, i.e., they sacrifice learning efficiency for the prestige. This can be used to explain the detrimental impact of affirmative action on its intended beneficiaries by overmatching them with far too challenging programs. However, among students with identical qualifications and hence identical loss in learning efficiency from an overmatch, it is the hardworking ones who suffer more in terms of final education outcomes, and they are more likely to stick to the optimal match while their less hardworking counterparts are more likely to opt for the overmatch. This self selection problem calls for a caveat in interpreting the empirical relationship established from observational data. Namely between two students with identical observable qualifications, even if data reveal that the one attending a more elite program thanks to affirmative action has lower education achievement that the one attending a less elite program, the gap cannot be entirely attributed to overmatch. Part of it may be due to overmatch and hence loss in the learning efficiency, but other part of it may be due to the self selection problem and hence unobserved heterogeneity in preference for leisure. Until the magnitude of the self selection problem can be determined relative to that of the overmatch, it should be cautioned to draw any causal relationship from affirmative action to overmatch in observational data.

In the existing literature, academic standard, ability tracking and school choice have been used to characterize other related aspects of education. When there lacks perfect information on the education achievement, or when the labor market is not fully competitive so that individuals are not paid by their own effective labor, Costrell (1994, 1997) and Betts (1998) model academic standard as a pass-it-or-fail-it threshold on the education outcome. All students essentially face the same education technology, and merely choose their efforts optimally. There is neither match nor overmatch. The discontinuity in the payoff profile on whether or not to meet the academic standard generates the discontinuity in the optimal effort among sufficiently similar students. In contrast, the current paper models academic standard as intrinsic to an education technology, and students first choose the curriculum that matches best with their qualifications, and then choose the optimal effort under the optimal curriculum. The discontinuity in school prestige from whether match or overmatch generates the discontinuity in the optimal effort among sufficiently similar students.

Another strand of the literature studies the effect of ability tracking in school finance and school choices. There the central assumption is the peer group effects, so that a high ability student generates positive externality on his peers, while a low ability student generates negative externality on his peers. Epple and Romano (1998, 2002), Epple, Newlon and Romano (2002) and Epple, Figlio and Romano (2004) analyze this issue in the setup of private versus public school choice. When students also differ in their income/wealth besides ability, and when private schools can use tuition as exclusion mechanism, it is shown that a hierarchy of school qualities can arise endogenously, where stratification along the income/wealth dimension is always the most prominent feature, and stratification along the ability dimension can also arise under certain circumstances. There all schools adopt the same education technology, and difference in school quality is purely determined by the composition of the student body and hence the mean ability level within a school or a track. Overmatch, if exists, is always beneficial in that a student enjoys better peers. In contrast, the central assumption of this paper is that schools adopt different education technologies that best suit students with different abilities. And to focus on the role of curriculum design, we abstract other issues such as school finance or peer group effects. A hierarchy of school qualities arises due to the technologies they adopt, and stratification along the ability dimension is the most prominent feature. Only in this framework can overmatch be of detrimental effect to a student, when the education technology is too challenging for his own ability.

Instead of using the market price to match students with private or public schools, an alternative mechanism is through a tournament. The tournament model can do away with the peer group effects and still generate stratification along the dimension of student ability. Fernandez and Gali (1999) compare the efficiency of matching students with schools through a market or through a tournament. The complementarity between student ability and school quality implies the socially efficient match is positive assortative, namely ability rank of a student is match to the quality rank of a school. Su (2005) analyzes this issue in a hierarchical education system, where student learning effort at the lower stage improves his qualification for the higher stage and also individual probability of getting admission therein. In both cases, from individual perspective, there is economic rent to be gained from either matching with a school of better quality or getting admission to higher education, so overmatch is always beneficial to the overmatched student. In the current paper, students of different abilities have comparative advantages in different curricula that suit them best, so that a detrimental effect can arise from overmatch.

Last but not the least, there is a small yet growing literature on vertically differentiated education technologies. Driskill and Horowitz (2002) and Su (2004, forthcoming) model education as a sequence of hierarchical stages, where the human capital output from a lower stage acts as an input in the education technology of a higher stage, and address issues like aggregate efficiency, distributional equality and the political economy of investment in different education stages. This paper abstract from the dynamic relationship in human capital accumulation across education stages, and instead model education as a one stage process with competing (horizontally differentiated) technologies. It is assumed that the value of attending a certain school can be decoupled into two parts: one productive in producing human capital, while the other called school prestige and purely psychological. So the optimal school choice based on the highest total value may deviate from the one based on the highest education value, and the impact of overmatch on education achievement can be readily analyzed.

The remaining part of the paper is organized as follows. Section 2 introduces the model of competing schools with heterogeneous curricula (education technologies). In section 3 equilibrium without school prestige is characterizes as the benchmark case, as the optimal school choice is based on the highest education value only. Then we analyze how students' optimal school choices change when there is school prestige, and link the results to current affirmative action debate. Section 4 discusses the robustness of the results with regard to the assumptions, and concludes that the case of the detrimental effect of affirmative action on its intended beneficiaries is weak at the best.

2 Schools with competing curricula

Consider an economy with a continuum of heterogeneous students and a finite number T of schools. Students differ in their qualification and preference for leisure, which are jointly distributed with the density function $\phi(.,.)$ on $[\underline{q}, \overline{q}] \times$

$[\underline{\theta}, \overline{\theta}]$. The density is assumed positive and finite everywhere, and total measure of students is normalized to be 1. Schools differ in their curriculum design (A_t, c_t), as well as total education expenditure G_t. For the current time, it is assumed that schools have no capacity constraints and admit any student that wants to attend. The effect of this assumption on our main results will be discussed in the next section.

For a student with qualification q and leisure preference θ, his human capital output from attending school t is given by the following education technology:

$$h_t = \begin{cases} 0 & \text{if } q \leq c_t \\ A_t(q - c_t)\frac{G_t}{N_t^{\alpha}} e_t(q, \theta) & \text{if } q > c_t \end{cases}$$

Three factors jointly contribute to the education outcome: student learning efficiency under the given curriculum $A_t(q - c_t)$, school quality $\frac{G_t}{N_t^{\alpha}}$, and student learning effort $e_t(q, \theta)$. The importance of curriculum design is captured in the tuple (A_t, c_t). The parameter c_t measures the minimum qualification requirement under this curriculum, below which a student cannot benefit from this type of schooling. A higher c_t implies a more challenging program and can also be interpreted as imposing a higher academic standard. Returning to our previous example of teaching intermediate microeconomics, it is obvious that a more stringent academic standard is imposed if the curriculum is to cover the Lagrange than it merely introduces the first order condition, and a student not so good in calculus may be totally lost in the former yet manages reasonably well in the latter.

On the other hand the parameter A_t measures the progress rate of the curriculum. A higher A_t implies faster progress during the schooling process. As can be easily seen, a higher A_t must accompany a higher c_t to guarantee that this education technology is not dominated in production efficiency. Without loss of generality, it is assumed that $\{A_t, c_t\}_{t=1}^{T}$ is ranked in a strictly ascending order, so that school 1 implements the curriculum with the lowest academic standard and the slowest progress rate, while school T implements the curriculum with the highest academic standard and the fastest progress rate.

It is worth pointing out that even though the curricula can be ranked by their academic standards and progress rates, they cannot be ranked when it comes to the production efficiency. Students with different qualifications benefit most from different curricula, so there exists an optimal match. It is equally harmful for student learning when the curriculum is too challenging or not challenging at all, while at the right level his learning efficiency $A_t(q-c_t)$ can be maximized. On one hand, high qualification students have absolute advantage in any given curriculum than low qualification students. On the other hand, high qualification students have comparative advantage in a more challenging curriculum, while low qualification students have comparative advantage in a less challenging curriculum. Each curriculum is the best choice for a stratum of the students with certain qualifications.

Abstracting from school quality, a student can choose among the curricula as follows. Consider two adjacent curricula t and $t+1$ where his qualification q meets both the minimum requirements. Then his learning efficiency is equal to $A_t(q-c_t)$ under curriculum t, and $A_{t+1}(q-c_{t+1})$ under curriculum $t+1$. It is easy to see that there exists $\widehat{q}_{(t+1)t} = \frac{A_{t+1}c_{t+1}-A_t c_t}{A_{t+1}-A_t}$ such that for $q < \widehat{q}_{(t+1)t}$, curriculum t suits him better and leads to higher learning efficiency; while for $q > \widehat{q}_{(t+1)t}$, curriculum $t+1$ suits him better and leads to higher learning efficiency. The student would be indifferent between the two curricula if $q = \widehat{q}_{(t+1)t}$.

To ensure that all curricula are at the technological frontier, it is further assumed that $\widehat{q}_{(t+2)(t+1)} > \widehat{q}_{(t+1)t}$. Otherwise curriculum $t+1$ is strictly dominated by curriculum t for $q_i < \widehat{q}_{(t+1)t}$, strictly dominated by curriculum $t+2$ when $q_i > \widehat{q}_{(t+2)(t+1)}$, and equivalent to both curricula t and $t+2$ in the case when $q_i = \widehat{q}_{(t+1)t} = \widehat{q}_{(t+2)(t+1)}$. Hence curriculum $t+1$ should never be adopted in the economy for efficiency concerns. It is also easy to check that the assumption $\widehat{q}_{(t+2)(t+1)} > \widehat{q}_{(t+1)t}$ is equivalent to $\widehat{q}_{(t+s)t} > \widehat{q}_{(t+1)t}$ for any $s > 1$, so that student preference over all curricula is single peaked.

For economic relevance, it is assumed that $c_1 < \underline{q} < \widehat{q}_{21}$ and $\overline{q} > \widehat{q}_{T(T-1)}$. The constraint on $\underline{q}$ states that even students with the lowest qualification can benefit from some curriculum, so that they are not excluded from education by curriculum design. Furthermore, students with the lowest qualification benefit more from curriculum 1 than any other curriculum, so that the least challenging curriculum is not dominated in production efficiency for the current population in the economy. The constraint on $\overline{q}$ states that students with the highest qualifications benefit more from curriculum T than any other curriculum, so that the most challenging curriculum is not dominated in production efficiency for the current population in the economy.

Besides curriculum design, school quality also affects education outcome. For simplicity, it is assumed that education is purely publicly financed without private education expenditure. Allowing the possibility of increasing returns to scale, school quality is determined by the total education expenditure G_t and the number of enrolled students N_t, with the parameter $\alpha \in (0,1]$ measuring the degree of increasing returns to scale. It is easy to see that $\alpha = 0$ implies schooling being a pure public good, while $\alpha = 1$ implies schooling being a pure private good. The complementarity in the education technology simply implies that for any given curriculum, a student always benefits more when there are better teachers and better facilities.

Taking school curricula and school quality $\{(A_t, c_t), \frac{G_t}{N_t^\alpha}\}_{t=1}^T$ as given, a student with qualification q and preference for leisure θ chooses the optimal school t, together with his optimal learning effort $e_t(q,\theta)$ for the following maximization problem:

$$\max A_t(q-c_t)\frac{G_t}{N_t^\alpha}e_t(q,\theta) + \theta\ln(1-e_t(q,\theta)) + u_t$$

where u_t is the exogenously given prestige value associated with school t, and $\{u_t\}_{t=1}^T$ is assumed weakly increasing for now. The interpretation and importance of u_t is to be discussed in detail in the next section.

Definition. *Let the subscript* i denote an individual student i. *An equilibrium consists of student school choice* $s_i \in \{1,2,...,T\}$ *and learning effort* $e_t(q_i,\theta_i)$*, together with school enrollment* $\{N_t\}_{t=1}^T$*, such that:*

(1) Given $\{N_t\}_{t=1}^T$, s_i *and* $e_t(q_i,\theta_i)$ *are individual optimal choices;*

(2) Consistency condition: $\{N_t\}_{t=1}^T$*is consistent with individual choice* s_i*, namely* $N_t = \int_{s_i=t} \phi(q,\theta)dqd\theta$.

3 Match or overmatch

The subscript i for an individual student is omitted when there is no risk of confusion. Individual optimization problem can be solved backwards. First, for a given school with curriculum (A_t, c_t) and quality $\frac{G_t}{N_t^\alpha}$, student optimal learning effort is determined as

$$e_t(q,\theta)^* = \begin{cases} 0 & \text{if } q \le c_t + \frac{\theta}{A_t\frac{G_t}{N_t^\alpha}} \\ 1 - \frac{\theta}{A_t(q-c_t)\frac{G_t}{N_t^\alpha}} & \text{if } q > c_t + \frac{\theta}{A_t\frac{G_t}{N_t^\alpha}} \end{cases}$$

With the optimal learning effort, the education value of school t for a student characterized by (q,θ) is thus

$$V_t(q,\theta) = A_t(q-c_t)\frac{G_t}{N_t^\alpha}e_t(q,\theta)^* + \theta\ln(1-e_t(q,\theta)^*)$$

It is obvious from the Envelope theorem that $\frac{dV_t(q,\theta)}{dq} \geq 0$, and $\frac{dV_t(q,\theta)}{dq} > 0$ for $q > c_t + \frac{\theta}{A_t \frac{G_t}{N_t^\alpha}}$, so high qualification students have absolute advantage in any given curriculum. Also $\frac{dV_t(q,\theta)}{d\theta} \leq 0$, and $\frac{dV_t(q,\theta)}{d\theta} < 0$ for $q > c_t + \frac{\theta}{A_t \frac{G_t}{N_t^\alpha}}$, so the education value of a school is decreasing in student preference for leisure. The total value of school t is given by $V_t(q,\theta) + u_t$, and the highest value among $\{V_t(q,\theta) + u_t\}_{t=1}^T$ determines the optimal school choice for a student characterized by (q,θ).

3.1 Without school prestige

In this subsection we characterize the equilibrium when there is no school prestige effect, namely $\{u_t\}_{t=1}^T$ is constant and normalized to 0 for all t. This is the benchmark case to determine the optimal match between students and schools, in that the optimal school choice is based on its education value only. When a given student chooses among schools, for the same learning effort, the marginal cost remains the same across schools, while the marginal benefit depends on school curriculum design and school quality. It can be easily seen that in this case, the optimal school choice depends only on his qualification q but not his preference for leisure θ.

Proposition 1 *If an equilibrium exists, denote* $\widetilde{q}_{(t+1)t} = \frac{A_{t+1}\frac{G_{t+1}}{N_{t+1}^\alpha}c_{t+1} - A_t \frac{G_t}{N_t^\alpha}c_t}{A_{t+1}\frac{G_{t+1}}{N_{t+1}^\alpha} - A_t \frac{G_t}{N_t^\alpha}}$, *then in the equilibrium* $\{\widetilde{q}_{(t+1)t}\}_{t=0}^T$ *is strictly increasing with* $\widetilde{q}_{10} = \underline{q}$ *and* $\widetilde{q}_{(T+1)T} = \overline{q}$.

This proposition characterizes the property of an equilibrium if it exists, which would in turn facilitate the proof of the existence and uniqueness of the equilibrium in the coming Proposition 3. It says that an equilibrium has to be such that none of the schools is dominated by other schools in terms of production efficiency. All schools adopt their curricula at the technological frontier, i.e., none of the curricula is dominated by other curricula in terms of production efficiency, then enrollments in all schools would adjust so that including school quality does not change the pattern. Alternatively speaking, when a positive measure of students find comparative advantage in any given curriculum, a positive measure of students would also find comparative advantage in any given school.

Proposition 2 *If an equilibrium exists, there is perfect stratification across* *schools along the dimension of students' qualifications, and the perfect stratification is independent of student preference for leisure.*

Like Proposition 1, this proposition also characterizes the property of an equilibrium if it exists. Without school prestige effect, optimal school choice is reduced to comparing the learning efficiency and school quality only, and independent of student preference for leisure θ. Perfect stratification along the q dimension dictates that students with low qualifications are optimally matched with schools adopting less challenging curricula, while students with high qualifications are optimally matched with schools adopting more challenging curricula. So the optimal match is also positive assortative as in Fernandez and Gali (1999). But unlike Fernandez and Gali (1999), here the match is both socially optimal and individually optimal, in that there is no economic rent to be gained for a student from an overmatch.

Proposition 3 *There exists a unique equilibrium with* $N_t > 0 \ \forall t$.

This proposition establishes the existence and uniqueness of the equilibrium when there is no school prestige. Since school enrollments are uniquely determined, so are school qualities. From Propositions 1 and 2, it is obvious that adding endogenously determined school quality does not alter the ranking of schools by how challenging their curricula are, endogenous school quality determination preserves the ranking. Each school serves a stratum of the student population with the best possible educations outcomes. And this is what we call (the optimal) match between students and schools.

3.2 With school prestige

Besides curricula, schools may also differ in their prestige. It is assumed to increase with the "eliteness" of a school, so that schools adopting more challenging curricula and hence more exclusive have higher prestige. In this sense $\{u_t\}_{t=1}^{T}$ is assumed to be strictly increasing with u_1 normalized to 0. Here School prestige may be interpreted in several ways. The simplest one is that it is purely psychological, so a student feels proud and derives utility from the relative standing of his qualification within the entire population. Alternatively, if the labor market is not perfectly competitive, frictionless or has perfect information, then school prestige can capture the benefit of network connections, name recognition, and even peer effects in that graduates from an elite school tend to have higher productivity on average. Last but not the least, school prestige may be interpreted as the peer effects in the schooling process, so that a student learns more with his better prepared classmates. In this paper we adopt the former two but exclude the third interpretation, so that school prestige can increase individual utility but has no productive role in enhancing one's education achievement. The impact of such an assumption on the results is to be discussed in the next section.

For clean comparison and easy interpretation of the results, when school prestige is added, we do not look at the equilibrium without capacity constraint. Instead we take the endogenously determined enrollment from the previous case without school prestige $\{N_t^*\}_{t=1}^T$ as given, and impose it as the capacity constraint on each school. This way school enrollment and hence school quality remains the same with and without school prestige. If a student changes his optimal school choice between these two cases, the change can be attributed to school prestige without ambiguity. Then the term overmatch refers to a student choosing a school that adopts a more challenging curriculum than the one yielding the highest education value. And the detrimental effect from an overmatch can be analyzed in a rational setup so that students sacrifice education achievements for school prestige.

When schools have capacity constraints, it is important to describe how they make their admission decisions. While student qualification is relatively easy to measure with sufficiently accurate tests, student preference for leisure is much harder to elicit. For simplicity, it is assumed that student qualification q is perfectly observable but preference for leisure θ is perfectly unobservable. Whether schools care for diversity or not, they want the student body with the highest qualification possible. So without affirmative action, they rank all applicants by their qualifications and start admission from the top till the capacity is filled. With affirmative action, they rank applicants by their qualifications within each group and start admission from the top till the group specific quota is reached. This is a fairly general specification that captures the essence of both explicit group quota and test score allowance.

On the other hand, a student may also prefer whether or not to attend a school different from the one he chooses when there is no school prestige. Since the original optimal match is the one yielding the highest education value, a student who changes his school choice when there is school prestige must incur loss in his education achievement. To compensate for that, he must gain in school prestige, namely it is an overmatch. An undermatch is never optimal in that it incurs both loss in education achievement and loss in school prestige.

Proposition 4 *If a student changes his school choice when there is school prestige from the optimal one when there is no school prestige, he must prefer an overmatch.*

On the other hand, whether a student actually prefers an overmatch to his original school choice now depends not only on his qualification q, but also on his preference for leisure θ. This is because his preference over schools hinges on whether the gain in school prestige from an overmatch is sufficient to offset his loss in education achievement. As clear from the previous analysis, two students with identical qualification q suffers the same loss in learning efficiency from an overmatch, yet the one with a higher preference for leisure θ invests less learning effort, so his total loss in education achievement is less severe than the one with a lower θ. Overall it is the hardworking students who are likely to stick to their original school choices, while the less hardworking students who are likely to prefer an overmatch. This causes the potential self selection problem in observational data.

Proposition 5 *If a student with* (q, θ) *prefers an overmatch when there is school prestige, then a student with the same* q *but* $\theta' > \theta$ *would also prefer the same overmatch; if a student with* (q, θ) *prefers the original school choice even when there is school prestige, then a student with the same* q *but* $\theta' < \theta$ *would also prefer the original school choice.*

Despite this single crossing property of student preference on match versus overmatch, student preference over all schools may be no longer single peaked, unlike the case without school prestige. If an student originally chooses school t as the optimal match, it is possible that with school prestige, the loss in education achievement from school $t + 1$ is greater than the gain in school prestige, yet when it comes to school $t + 2$, the loss in education achievement is more than compensated by the gain in school prestige. One specific situation is that the curriculum in the overmatch is so challenging that student learning effort is reduced to 0, namely he is only enjoy the prestige value but not the education value from schooling. In this case a yet further overmatch can only be more desirable, in that he has nothing to lose in terms of education achievement but everything to gain from school prestige. This is especially true for student wit very low qualification or very high preference for leisure.

Proposition 6 *If a student with* (q, θ) *prefers an overmatch to his original optimal school choice, and in this overmatch his learning effort is 0, then his most preferred school is school* T *, and so is it for any student with* (q', θ') *such that* $q' \leq q$ *and/or* $\theta' > \theta$*.*

Overall if schools adopt affirmative action in their admission policies, the effect would be to grant admission to some students with lower qualifications who would otherwise only get admission from less prestigious schools. But who actually accept the newly available admission from overmatched schools would be a self selection problem. Propositions 5 and 6 show that overmatch is more beneficial for students with higher preference for leisure; or in the case with corner solution for learning effort, it does not differentiate among students with high preference for leisure and/or low qualification. This self selection problem widens the gap in observed education achievements between students switching for the overmatch and those sticking to their original optimal school choices, so that wee have to be cautious to interpret the empirical relationship from observational data.

4 Discussion and conclusion

This paper builds a simple theoretical model to study the effect of curriculum design and school prestige on student school choice. More specifically it analyzes the possibility of a detrimental effect of affirmative action on its intended beneficiaries by overmatching them with schools. In the analysis several simplifying assumptions are made, some less controversial than others. In this section we discuss the robustness of the main results with respect to these assumptions

One assumption in the benchmark case is that schools have unrestricted capacity, so that students can attend whichever school that is best suited for them to obtain the highest education achievements. This assumption is most conducive to the detrimental effect of an overmatch, in that by definition an overmatch incurs loss in education achievement. If instead schools have restricted capacity, for example minority students cannot attend their optimal schools without affirmative action but can with affirmative action, then an overmatch would generate double benefit in both the education achievement and school prestige. As shown in Su (2005), when there is economic rent to be gained for the minority students from better education opportunities, affirmative action may leads to better education achievement not only for its intended beneficiaries, but also for the majority students due to competition among students. The detrimental effect of affirmative action on its intended beneficiaries is less likely to arise when schools initially impose capacity constraints.

A related assumption is that education is a one stage process and student qualification is exogenously given. Alternatively, following Su (2005), education can be modeled as a multi-stage process, and student qualification for a higher stage may be augmented by his learning effort at the lower stage. Then the possibility of an overmatch at the higher stage that would not otherwise exist implies a better education opportunity, which may in turn induce more learning effort from minority students at the lower stage, so that they can better take advantage of the newly available opportunity. The detrimental effect of affirmative action on its intended beneficiaries is less likely to arise when student qualification is endogenously determined from his previous schooling process.

A major deviation of the current paper from the existing literature on education tracks is that no peer group effect is assumed in the education technology. Again this is more conducive to the detrimental effect of an overmatch. If peer group effect is present in the education technology, the benefit from better classmates can at least offset some of the detrimental impact from too challenging a curriculum, and the loss in student education achievement from an overmatch would not be as severe, if at all.

Needless to say, if the value of attending a school goes beyond its education value, then the presence of school prestige may lead students to choose schools other than the ones generating the highest education achievements for them, and hence the possibility of the detrimental impact of affirmative action on its intended beneficiaries by overmatching them with schools. First the rationality assumption implies that overmatched students enjoy higher utility levels than otherwise. and second, even if we focus our attention to the education achievement only, and even with the assumptions that are most conducive to the detrimental impact of an overmatch, we show that the self selection problem among students whether or not to accept an overmatch can widen the their education achievement gap due to unobserved heterogeneity in preference for leisure. So the case of the detrimental impact of affirmative action on its intended beneficiaries is weak at the best, and we should be cautioned in drawing causal relationship from observational data.

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Appendix

Proposition 1.

Proof. Suppose there exists a t such that $\widetilde{q}_{(t+2)(t+1)} \leq \widetilde{q}_{(t+1)t}$, *then school* $t+1$ is strictly dominated by school t when $q < \widetilde{q}_{(t+1)t}$, *strictly dominated by school* $t+2$ when $q > \widetilde{q}_{(t+2)(t+1)}$, and weakly dominated by both schools t and $t+2$ in the case $q = \widetilde{q}_{(t+1)t} = \widetilde{q}_{(t+2)(t+1)}$. Hence $N_{t+1} = 0$. Yet with $N_{t+1} = 0$, $\widetilde{q}_{(t+1)t} = c_{t+1}$ and $\widetilde{q}_{(t+2)(t+1)} > c_{t+2}$, contradiction. ■

Proposition 2.

Proof. From Proposition 1, if an equilibrium exists, $\{\widetilde{q}_{(t+1)t}\}_{t=0}^{T}$ is strictly increasing, so for $q \in [\widetilde{q}_{(t+2)(t+1)}, \widetilde{q}_{(t+1)t}]$ for $t \in \{0, 1, ..., T-1\}$, the optimal choice is school $t+1$. The perfect stratification of students across schools is independent of θ_i. ■

Proposition 3.

Proof. Kakutani's fixed point theorem will be applied to establish the existence of an equilibrium. Denote $\mathbf{N} = (N_1, N_2, ..., N_T)' \in R_+^T$ and Δ the unit simplex in R^T. Obviously Δ is a nonempty, convex and compact set. The mapping $\Xi : \Delta \rightarrow \Delta$ is defined as follows:

For $\mathbf{N} \in \Delta^o$ ($\mathbf{N} \gg 0$), it is straight forward to compute $\{\widetilde{q}_{(t+1)t}\}_{t=0}^{T}$ as in Lemma 1 for $t \in \{1, 2, ..., T-1\}$, and $\widetilde{q}_{01} \equiv \underline{q}$ and $\widetilde{q}_{(T+1)T} \equiv \overline{q}$. Thus from Lemma 2 $\Xi(\mathbf{N})_t = \int_{\widetilde{q}_{t(t-1)}}^{\widetilde{q}_{(t+1)t}} \int_{\underline{\theta}}^{\overline{\theta}} \phi d\theta dq$, and since ϕ is the density function on $[\underline{q}, \overline{q}] \times [\underline{\theta}, \overline{\theta}]$, it is obvious that $\sum_{t=1}^{T} \Xi(\mathbf{N})_t = 1$, or $\Xi(\mathbf{N}) \in \Delta$.

For $\mathbf{N} \in \partial\Delta$, let $\{k_1, k_2, ..., k_T\}$ be a rearrangement of $\{1, 2, ..., T\}$ such that $\{k_1, k_2, ..., k_s\}$ is strictly increasing and $N_r > 0$ for $r \in \{k_1, k_2, ..., k_s\}$, and $\{k_{s+1}, k_{s+2}, ..., k_T\}$ is strictly increasing and $N_r = 0$ for $r \in \{k_{s+1}, k_{s+2}, ..., k_T\}$. For $r \in \{k_1, k_2, ..., k_s\}$, if $r > k_{s+1}$, then $\Xi(\mathbf{N})_r = 0$; if $r < k_{s+1}$ and $r + 1 = k_{s+1}$, then $\Xi(\mathbf{N})_r = \int_{\widetilde{q}_{r(r-1)}}^{c_{r+1}} \int_{\underline{\theta}}^{\overline{\theta}} \phi d\theta dq$; if $r < k_{s+1}$ and $r + 1 < k_{s+1}$, then $\Xi(\mathbf{N})_r = \int_{\widetilde{q}_{r(r-1)}}^{\widetilde{q}_{(r+1)r}} \int_{\underline{\theta}}^{\overline{\theta}} \phi d\theta dq$. For $r \in \{k_{s+1}, k_{s+2}, ..., k_T\}$, if $r = k_{s+1}$, then $\int_{c_{k_{s+1}}}^{c_{k_{s+2}}} \int_{\underline{\theta}}^{\overline{\theta}} \phi d\theta dq \leq \Xi(\mathbf{N})_r \leq \int_{c_{k_{s+1}}}^{\overline{q}} \int_{\underline{\theta}}^{\overline{\theta}} \phi d\theta dq$; and if $r > k_{s+1}$, then $0 \leq \Xi(\mathbf{N})_r \leq \int_{c_r}^{\overline{q}} \int_{\underline{\theta}}^{\overline{\theta}} \phi d\theta dq$; and $\sum_{t=1}^{T} \Xi(\mathbf{N})_t = 1$, or $\Xi(\mathbf{N}) \in \Delta$.

Overall Ξ is a convex-valued, upper hemicontinuous correspondence from Δ to Δ, and hence there exists a $\mathbf{N}^* \in \Delta$ such that $\Xi(\mathbf{N}^*) = \mathbf{N}^*$. By the construction of Ξ, both equilibrium conditions are satisfied, so $\mathbf{N}^*$ is the equilibrium solution. From Proposition 1, $\mathbf{N}^* \gg 0$.

For the uniqueness, suppose there are two different equilibrium solutions $\{N_t\}_{t=1}^T$ and $\{N_t'\}_{t=1}^T$. Let s be the smallest number such that $N_s \neq N_s^{'}$. Without loss of generality, let $N_s > N_s'$. If $s > 1$, $N_r = N_r'$ for $r \leq s-1$ and $N_s > N_s'$ implies that $\widetilde{q}_{(s-1)(s-2)} = \widetilde{q}'_{(s-1)(s-2)}$ and $\widetilde{q}_{s(s-1)} > \widetilde{q}'_{s(s-1)}$, yet this in turn implies that $N_{s-1} > N_{s-1}^{'}$, which is a contradiction. Next if $s = 1$, $N_1 > N_1'$ implies that $\widetilde{q}_{21} > \widetilde{q}'_{21}$, yet for the marginal student to be indifferent between school 1 and 2, it has to be the case that $N_2 > N_2'$, and so forth. So we have $N_t > N_t' \forall t$, yet $\sum_{t=1}^T N_t = \sum_{t=1}^T N_t' = 1$, which is a contradiction. So there is a unique equilibrium. ■

Proposition 4.

Proof. By definition, if the optimal school choice is school t without school prestige, then $V_t(q,\theta) \geq V_s(q,\theta)$ and $u_t > u_s$ for $t > s$, so school s cannot be the optimal choice with school prestige. Hence if the optimal school choice changes, it can only be an overmatch. ■

Proposition 5.

Proof. For two students characterized by (q,θ) and (q,θ'), without school prestige, both share the same optimal choice of school t. With school prestige, if student (q,θ) now prefers school $s > t$, then $V_t(q,\theta) + u_t < V_s(q,\theta) + u_s$, or equivalently, $V_t(q,\theta) - V_s(q,\theta) < u_s - u_t$. Applying the Envelope Theorem, it is easy to check that $\frac{d(V_t(q,\theta) - V_s(q,\theta))}{d\theta} = \ln(1 - e_t(q,\theta)^*) - \ln(1 - e_s(q,\theta)^*) < 0$, where the inequality follows from the definition of the optimal school choice without school prestige and hence $e_t(q,\theta)^* > e_s(q,\theta)^*$. Then for $\theta' > \theta$, it follows that $V_t(q,\theta') - V_s(q,\theta') < V_t(q,\theta) - V_s(q,\theta) < u_s - u_t$., so student (q,θ') also prefers the same overmatch. Parallel argument proves the second half of the proposition. ■

Proposition 6.

Proof. Suppose the student prefers school $s > t$ to school t and $e_s(q,\theta)^* = 0$, then $V_t(q,\theta) + u_t < u_s$. Without school prestige, the original optimal school t is such that $e_t(q,\theta)^* > 0$, so by the Envelope Theorem $\frac{dV_t(q,\theta)}{dq} > 0$ and $\frac{dV_t(q,\theta)}{d\theta} < 0$. It is then obvious that $V_t(q',\theta') + u_t < u_s$ for any $q' \leq q$ or $\theta' \geq \theta$. The strict increasing $\{u_t\}_{t=1}^T$ implies that the most preferred is school T. ■