Education Hierarchy, Within-Group Competition and Affirmative Action

Xuejuan Su*

January 2005

Abstract

This paper studies the impact of Affirmative Action in a hierarchical education framework, explicitly taking students competition for higher education opportunity into account. Initial racial inequality translates into inferior development opportunity for students in the disadvantaged group, thus higher education admission based purely on test score ranking is actually discriminatory against the historically discriminated. Without policy intervention, hierarchical education exacerbates racial inequality. Affirmative Action improves racial equality, and need not invoke efficiency loss. When initial competition is too low among students, Affirmative Action can increase competition and lead to more effort and higher human capital achievements for all students.

JEL code: I28, J78, H42

Keywords: Affirmative Action, Hierarchical education, Competition

*Author address: Department of Economics, Finance and Legal Studies, University of Alabama, Tuscaloosa, AL 35487-0224. Email address: xsu@cba.ua.edu.

1. Introduction

In the United States, Affirmative Action has been adopted to improve social equality since the Civil Rights movement. The policy effect on social equality as well as that on aggregate efficiency has been the focus of a small yet growing literature.[1] Within the labor market, Schotter and Weigelt (1992) uses a two-player asymmetric tournament model to show that Affirmative Action giving preferential treatment to one player always comes at the expense of the other player, so their optimal efforts change in opposite directions, and the aggregate effect depends on the magnitude of initial asymmetry. Based on the statistical discrimination models initiated by Arrow (1973) and Phelps (1972), Coate and Loury (1993) shows that Affirmative Action may have ambiguous effect in eliminating the negative stereotype. In certain cases it induces more effort and hence better qualification from the favored group; yet in other cases it has the opposite effect when employers are induced to patronize the under-qualified employees from the favored group. Chung (1999) models this patronization as an implementation problem within the mechanism design framework. He shows that more complicated forms of Affirmative Action can selectively eliminate the undesirable equilibria associated with simple forms of Affirmative Action in the labor market.

Another strand studies the policy effect within the education framework. Chan and Eyster (2003) shows that explicit Affirmative Action is the most efficient way to achieve racial diversity. When the distribution of qualification differs across racial groups, race-conscious Affirmative Action can select the best-qualified students within each group, while race-blind admission policies achieve racial diversity at the cost of lower entering student quality. De Fraja (forthcoming) also shows that Affirmative Action is an efficient form to allocate public education subsidy. Given the tax distortion associated with public education subsidy, if the

[1] See Holzer and Neumark (2000) for an excellent survey of this literature.

government has only information on group distribution but not individual qualification, it is less costly to give preferential treatment to the disadvantaged group since it has fewer individuals with high qualifications.

The current paper distinguishes from the existing literature in two aspects. First, relevant to both the labor market and the education process, within-group competition plays a crucial role yet has been largely overlooked in the literature. When within-group competition is absent, as in the two-player setup in Schotter and Weigelt (1992), preferential treatment to one player always comes at the expense of the other player, so Affirmative Action has opposite effect on the two players' optimal efforts, and hence always involves efficiency trade-off. Yet in reality most Affirmative Action policies apply to a disadvantaged group instead of a single individual. Within-group competition thus arises when individuals in the disadvantaged group compete for the potential benefit, and individuals in the advantaged group compete to avoid the potential cost associated with Affirmative Action. Adding within-group competition into the analysis allows the possibility that individuals in both groups increase their efforts when Affirmative Action introduces more competition, despite the fact that one group's improved opportunity always comes at the expense of the other group's worsened opportunity. Thus Affirmative Action need not invoke efficiency trade-off between groups, and consequently may improve social equality without hurting aggregate efficiency.

The other distinguishing feature of the current paper is specific to the education process, which is intrinsically hierarchical. Students have to finish basic education successfully before attending higher education. More importantly, their academic achievements at basic education improves their learning efficiency at higher education. Put in a tournament model, student effort at basic education affects not only the probability of winning the tournament, i.e., getting higher education admission, but also the size of the prize, i.e., the value of the higher education opportunity. Yet this dynamic relationship in human capital accumulation across education stages has again be overlooked in the literature, as the distribution of student qualification (and learning efficiency) is assumed exogenous and invariant when Affirmative Action is considered, as in Chan and Eyster (2003) and De Fraja (forthcoming). Allowing individual response to Affirmative Action and hence endogenizing the distribution of qualification lends extra support to the justification of Affirmative Action, which has faced a series of attacks recently. With education hierarchy, existing social inequality translates into inferior development opportunity for children within the disadvantaged group, so higher education admission based purely on test score ranking is actually discriminatory against the historically discriminated and exacerbates social inequality. Affirmative Action, on the other hand, serves as an effective means to redress problems resulting from historical discrimination, and improve social equality.[2]

This paper studies the policy effect of Affirmative Action in a hierarchical education framework, explicitly taking student competition for higher education opportunity into account. The two-stage education model follows directly from Su (2004a, 2004b), which proves extremely useful when analyzing education policies at a specific stage in the education process. In this model there is no fundamental difference between racial groups, except for the gap in initial human capital endowment resulting from historical discrimination. Without policy intervention, limited access to higher education based purely on test score ranking exacerbates racial inequality. With policy intervention, Affirmative Action improves racial inequality. Furthermore, the goal of improving racial equality need not be at conflict with improving aggregate efficiency. When higher education capacity is not too limited, and when initially there is little competition among students, Affirmative Action can introduce more competition, induce more efforts and lead to higher

[2]For court rulings on the conditions under which Affirmative Action is not unconstitutional, see *Regent of the University of California v. Bakke, Podberesky v. Kirwan, Hopwood v. State of Texas* and the two recent Michigan cases *Grutter v. Bollinger* and *Gratz v. Bollinger.*

human capital achievements for all individuals in both groups. Two forms of Affirmative Action are considered, where the policy effect differs only quantitatively but not qualitatively.

The rest of the paper is structured as follows. Section 2 introduces the two-stage education model. The equilibrium outcome without Affirmative Action is studied in section 3. Section 4 analyzes the impact of one form of Affirmative Action in higher education admission, namely a certain number of college seats are reserved for the disadvantaged group. Analytical results together with some numerical examples are characterized. As robustness test, another form of Affirmative Action is examined in section 5, where test scores are adjusted for students from the disadvantaged group. Section 6 draws the conclusion. All proofs and figures are in the Appendix.

2. Two-stage education model

This is a two-generation model with two groups.[3] Within each group there is a continuum of homogeneous individuals. We can call these two groups Red and Green, or we can call them Black and White. There is no fundamental difference between the two groups. However, due to historical discrimination, the black group did not have as good development opportunity as the white group. As a result, the black group has lower human capital endowment than the white group, namely $H_{-1}^{B} < H_{-1}^{W}$, with the superscript $r \in \{B,\ W\}$ denoting the group and the subscript -1 denoting the parent generation. From now on all capital letter variables represent group-wide average, with their small letter counterparts representing individual choices within a group. Also normalize the population size to 1, and the two groups are of measure λ and $1 - \lambda$ respectively.

[3]This two-generation model can be easily extended to infinite horizon and hence infinite generations recursively.

A child accumulates human capital through two stages of education. Basic education is provided locally according to the following technology, with the superscript 1 denoting the basic education (first) stage:

$$h^{r,1} = AH^{r}_{-1}e^{r,1} \tag{1}$$

The basic education technology is pretty standard. It has two inputs: H^{r}_{-1} the average human capital level of the parents within the child's own group; and $e^{r,1}$ the child's learning effort at this stage.[4] Empirical evidence shows that depending on their socioeconomic background, different racial groups self select into different communities, which differ drastically in their schooling quality. This de facto segregation implies that black students face inferior development opportunity compared to their white peers.[5] The group-wide average H^{r}_{-1} is thus used to capture the "community effect".

After finishing basic education, a student may have the opportunity of attending higher education, which has the following education technology, with the superscript 2 denoting the higher education (second) stage:

$$h^{r,2} = Bh^{r,1}e^{r,2} \tag{2}$$

The higher education technology explicitly captures the dynamic relationship in human capital accumulation. It parallels that in basic education and also has

[4]For all subsequent analysis, it makes no difference whether we use the group-wide average H^{r}_{-1} or the individual parent human capital h^{r}_{-1} as an iuput. It only makes a difference when the two-generation model is extended to infinite horizon and when ex ante homogeneous individuals within a group become ex post heterogenous depending on whether they get higher education admission or not. It then is much easier to adopt the current specification for tractability of the state space overtime.

[5]See Currie and Thomas (1995), Neal and Johnson (1996), and Rogers and Spriggs (1996), among others, for the impact of inferior family background and neighborhood characteristics on children's academic performance.

two inputs: $h^{r,1}$ the child's qualification for higher education and $e^{r,2}$ his learning effort at this stage. That the learning efficiency in higher education depends on the human capital output from basic education is intrinsic to the hierarchical education process. Also for simplicity, it is assumed that higher education does not depend on group characteristics directly, although indirectly group characteristics enter through the education hierarchy.

While basic education is universally available, higher education has capacity constraint. More specifically, out of the entire population, there is only a fraction Σ, with $0 < \Sigma < 1$, that can get higher education admission. As a result, a student perceives his probability σ^r of attending higher education (the determination of which is given by the admission policy and to be discussed later), and he chooses $e^{r,1}$ and $e^{r,2}$ to maximize his expected utility:

$$h^{r,1} + \theta_1 \ln(1 - e^{r,1}) + \sigma^r (h^{r,2} + \theta_2 \ln(1 - e^{r,2})) \tag{3}$$

With or without Affirmative Action, the admission policy is always meritocratic. It is based on students' test scores in the matriculation exam; and within the properly defined sub-population for consideration, those with higher scores are admitted before those with lower scores. However, the test score is only an imperfect measure of the human capital output from basic education:

$$s^r = h^{r,1} + \varepsilon \tag{4}$$

where ε is a mean zero random variable, independently and identically distributed across individuals, with the cumulative distribution function $G(\varepsilon)$ and the probability density function $g(\varepsilon) = G'(\varepsilon)$.

Definition *Given an admission policy, for $r \in \{B, W\}$, a symmetric equilibrium consists of individual learning effort in basic and higher education $\{e^{r,1}_{t,i}, e^{r,2}\}$,* *individual perceived probability of higher education σ^r, together with the group-wide learning effort in basic and higher education $\{E^{r,1},\ E^{r,2}\}$, and group-wide probability of higher education Σ^r, such that:*

(1) Given σ^r, $e^{r,1}$ and $e^{r,2}$ maximize the individual's expected utility;

(2) Given $E^{B,1}$ and $E^{W,1}$, σ^r is consistent with the admission policy for $r \in \{B,\ W\}$;

(3) Symmetry: $e^{r,1} = E^{r,1}$, $e^{r,2} = E^{r,2}$ and $\sigma^r = \Sigma^r$;

(4) Capacity constraint: $\lambda\Sigma^B + (1-\lambda)\Sigma^W = \Sigma$.

3. No Affirmative Action

As a benchmark, when there is no higher education in the economy, or equivalently, when a student perceives zero probability in both attending higher education and in improving his probability of attending higher education, his optimization problem is reduced to the choice $e^{r,1}$ only, with the first order condition given by:

$$AH^r_{-1} = \frac{\theta_1}{1-e^{r,1}} \tag{5}$$

Assumption 1. $AH^B_{-1} > \theta_1$

The Inada condition on leisure guarantees that $e^{r,1}$ is bounded below 1. Assumption 1 implies that $AH^W_{-1} > AH^B_{-1} > \theta_1$, so $e^{r,1}$ as the solution to (5) is also bounded above 0 and is an interior solution. This assumption simply says that at the basic education stage, leisure is not valued too much compared to learning and hence consumption, so that individuals find it worthwhile to invest in human capital accumulation.

When a student perceives positive probability of attending higher education, he can always wait to choose $e^{r,2}$ after the uncertainty of whether he actually gets higher education admission or not is resolved. Consequently the first order condition on $e^{r,2}$ is deterministic and independent of the perceived probability σ^r:

$$e^{r,2*} = 1 - \frac{\theta_2}{ABH_{-1}^r e^{r,1}} \tag{6}$$

Assumption 2. $B(AH_{-1}^B - \theta_1) > \theta_2$

Again the Inada condition on leisure guarantees that $e^{r,2}$ is bounded below 1. Assumption 2 implies that $B(AH_{-1}^W - \theta_1) > B(AH_{-1}^B - \theta_1) > \theta_2$, so $e^{r,2*}$ is bounded above 0 when $e^{r,1} = \underline{e}^{r,1} \equiv 1 - \frac{\theta_1}{AH_{-1}^r}$ from (5). As will become clear in a moment, individual optimal choice of $e^{r,1}$ is always at least as high as $\underline{e}^{r,1}$, so $e^{r,2*}$ is always an interior solution.

It is obvious from (6) that individual optimal effort in higher education $e^{r,2*}$ depends positively on his effort in basic education $e^{r,1}$. This positive dependence reflects the dynamic relationship in human capital accumulation through education hierarchy. When a student works hard in basic education, he accumulates more human capital and is better prepared for higher education. This better learning efficiency in turn enhances the rate of return on higher education and leads to more effort.

As can be easily seen, higher education has zero value when an individual invests zero effort in it. With the optimal effort $e^{r,2*} > 0$, the value of higher education becomes positive:

$$v^{r,2}(e^{r,1}) = ABH_{-1}^r e^{r,1} e^{r,2*} + \theta_2 \ln(1 - e^{r,2*}) > 0 \tag{7}$$

The net benefit of higher education induces students to work hard and compete for the limited opportunity by out-performing others in the exam. This is the source of competition.

Assumption 3a. $\varepsilon \sim dist.[\underline{\varepsilon},\ \overline{\varepsilon}]$ such that $AH_{-1}^B + \overline{\varepsilon} < AH_{-1}^W + \underline{\varepsilon}$

Under Assumption 3a, the random variable ε affecting the test score has a bounded support $[\underline{\varepsilon},\ \overline{\varepsilon}]$. Moreover, the support is small relative to the initial racial gap in human capital. If individuals choose $e^{r,1} = \underline{e}^{r,1}$ in basic education, then Assumption 3a implies that $AH_{-1}^{B} - \theta_1 + \overline{\varepsilon} < AH_{-1}^{W} - \theta_1 + \underline{\varepsilon}$, namely even the most lucky black student has a lower score than the least lucky white student, and there is perfect stratification between the two groups.[6] As will become clear in a moment, a positive probability of higher education for white but not black students will magnify the gap in their optimal efforts and hence test scores, so that perfect stratification between the two groups remains. In the benchmark case without Affirmative Action, this perfect stratification assumption rules out between-group competition, and allows us to focus on the within-group competition for the limited higher education opportunity.

Within-group competition arises when, taking other students' decisions as given, a student can increase his own chance of getting higher education admission by investing more effort in basic education and getting better test score. Without Affirmative Action, perfect stratification implies that the limited capacity for higher education is filled first by white students only. With little loss of generality, it is assumed that $\Sigma < 1 - \lambda$,[7] so that $\Sigma^W = \frac{\Sigma}{1-\lambda} < 1$, and there is within-group competition for higher education among white students.

When the continuum of all other white students choose the learning effort $E^{W,1}$ in basic education, they have identical human capital output from basic education $H^{W,1} = AH_{-1}^{W} E^{W,1}$. Since ε is independently and identically distributed, test scores of all other white students follow the distribution $H^{W,1} + \varepsilon$. Thus the cut-off test score for higher education admission is given by:

$$\widehat{s} = H^{W,1} + \widehat{\varepsilon} = H^{W,1} + G^{-1}(1 - \frac{\Sigma}{1-\lambda}) \tag{8}$$

[6] This condition can also be interpreted as that the matriculation exam can always be designed to a sufficient precision level so that there is perfect stratification between the two groups. Chan and Eyster (2002) studies individual preferences over matriculation exams with different precision levels.

[7] Parallel analysis applies to the case when $\Sigma \geq 1 - \lambda$.

Taking the test score distribution as given, a white student with learning effort $e^{W,1}$ perceives his own probability of getting higher education admission as

$$\sigma^W = \Pr(h^{W,1} + \varepsilon \geq \widehat{s}) = 1 - G(H^{W,1} - h^{W,1} + \widehat{\varepsilon}) \tag{9}$$

Hence the within-group competition is represented by the positive marginal effect of investing more effort in basic education on improving his own probability σ^W:

$$\frac{\partial \sigma^W}{\partial e^{W,1}} = g(H^{W,1} - h^{W,1} + \widehat{\varepsilon})AH^W_{-1} > 0 \tag{10}$$

In a symmetric equilibrium $h^{W,1} = H^{W,1}$, so $\sigma^W = \Sigma^W$ and $\frac{\partial \sigma^W}{\partial e^{W,1}} = g(\widehat{\varepsilon})AH^W_{-1}$, with the marginal effect increasing with the degree of within-group competition measured by the probability density at the cut-off point $g(\widehat{\varepsilon})$.

Utilizing (6) and 7), now the first order condition on $e^{W,1}$ is given by the following:

$$AH^W_{-1} + \sigma^W ABH^W_{-1}e^{W,2*} + \frac{\partial \sigma^W}{\partial e^{W,1}} v^{W,2}(e^{W,1}) = \frac{\theta_1}{1 - e^{W,1}} \tag{11}$$

This is the central equation of the model, which we will return to frequently in the subsequent analysis of the policy impact of Affirmative Action. Intuitively, the right-hand side term $\frac{\theta_1}{1-e^{W,1}}$ represents the marginal cost of learning effort in basic education, which is the foregone leisure. The left-hand side represents the marginal benefit, which can be decomposed into three distinct terms. The first term AH^W_{-1} reflects how the learning effort in basic education produces human capital in its own name, which contributes to wage income regardless of the higher education opportunity. The second term $\sigma^W ABH^W_{-1}e^{W,2**}$ shows how with probability σ^W, the learning effort in basic education improves the learning efficiency in higher education, and hence contributes to the human capital output and wage income. The third term $\frac{\partial \sigma^W}{\partial e^{W,1}} v^{W,2}(e^{W,1})$ captures the within-group competition effect, where putting more effort in basic education increases individual probability of getting higher education admission by $\frac{\partial \sigma^W}{\partial e^{W,1}}$, the value of which is $v^{W,2}(e^{W,1})$.

While the marginal cost curve is increasing and convex, the marginal benefit curve may be either increasing or decreasing. As can be easily checked, when $e^{W,1} = \underline{e}^{W,1}$, marginal benefit is bigger than marginal cost; yet when $e^{W,1}$ is sufficiently close to 1, marginal cost is bigger than marginal benefit. Hence the marginal benefit curve cuts the marginal cost curve from above at least once, which produces a local maximum. It is assumed that the two curves cross once and only once, so the unique local maximum is also the global maximum.

As for a black student, perfect stratification implies that $\sigma^B = \Sigma^B = 0$ and $\frac{\partial \sigma^B}{\partial e^{B,1}} = 0$. So his optimal choice is $e^{B,1} = \underline{e}^{B,1}$. Compared to (11), while the marginal cost curve for a black student remains the same as that for a white student, the marginal benefit curve shifts downward due to the two missing components associated with higher education. Consequently in equilibrium $e^{B,1*} = \underline{e}^{B,1} < \underline{e}^{W,1} < e^{W,1*}$. Overall the lack of higher education opportunity hurts black students not only in the missing higher education stage, but also in the basic education stage by repressing their optimal learning efforts.

Proposition 1. *With inferior development opportunities for black students in basic education, limited access to higher education without Affirmative Action exacerbates the racial inequality.*

Proof. *See the Appendix.* ■

This Proposition lends extra support to the (legal) justification for Affirmative Action in higher education, based on the intrinsic hierarchy in the education process. Initial racial inequality resulting from historical discrimination translates into inferior development opportunity for black students at the basic education stage. As a result, the seemingly fair admission policy at higher education, based purely on test score ranking but not racial identity, is actually discriminatory against the historically discriminated and exacerbates racial inequality.

If policy intervention is called for to redress this problem, the least controversial practice would be to "level the playing field". As Steele (1990) writes, "Give my children fairness; give disadvantaged children a better shot at development - better elementary and secondary schools, ..., safer neighborhoods, ..., and so on (pp. 124)." In the terminology of the current model, the economy-wide average human capital level of the parent generation would replace the group-specific variables at the basic education stage. However, "leveling the playing field" implies sizeable expenditure from the central government to improve schooling quality at poorer communities, which may not be fiscally feasible. On the other hand, Affirmative Action in higher education admission incurs little explicit cost, and simply redistributes some higher education opportunity from the white group to the black group. In the next two sections we will focus on the potential implicit cost of Affirmative Action measured by the efficiency loss in individual and aggregate human capital accumulation.

4. Affirmative Action I: college seats allotment

In this section, we consider one form of Affirmative Action in higher education admission policy, when a fixed number of college seats is reserved for the disadvantaged group.[8] Essentially this admission policy split the total higher education opportunity into two group-specific opportunities Σ^B and Σ^W, and allows students to compete only with their peers in the same group. Within each group, students are ranked by their test scores, and those with higher scores get admission up to the allotted group seats. Under this policy between-group competition is again excluded, and we can conveniently focus on the role of within-group competition.

Compared to the benchmark case without Affirmative Action where Σ^B =

[8] Even though this practice has been ruled unconstitutional in the *Bakke* case, it provides a good starting point in understanding the efficiency impact of Affirmative Action..

0 and $\Sigma^W = \frac{\Sigma}{1-\lambda}$, now Affirmative Action in higher education admission sets $\Sigma^B > 0$ and $\Sigma^W = \frac{\Sigma - \lambda\Sigma^B}{1-\lambda}$ satisfying the capacity constraint. Affirmative Action increases the higher education opportunity for the black group at the expense of the white group. Yet with within-group competition, the change in the group-wide probability does not directly translate into corresponding change in individual perceived probability of higher education, and hence optimal learning effort at basic education.

Proposition 2. *Affirmative Action, which increases the higher education opportunity for black students from* $\Sigma^B = 0$ *to* $\Sigma^B > 0$*, leads to higher learning effort and higher human capital achievement for all black students.*
Proof. *See the Appendix.* ■

It is natural to expect that Affirmative Action improves the human capital achievement for those "lucky" black students who could not otherwise get higher education admission. But the effect of Affirmative Action extends beyond that. It induces higher learning effort at basic education for all black students, even if ex post they do not attend higher education. Due to the dynamic relationship in human capital accumulation through the education hierarchy, a positive probability of higher education per se increases the expected return on learning effort at basic education, in that it can improve the learning efficiency when a black student does get higher education admission. Moreover, the positive net benefit of higher education induces all black students to compete for the opportunity by investing more effort in basic education and outperforming others in the exam. In the symmetric equilibrium all black students end up choosing the same effort level, but it is significantly higher with the presence of within-group competition than its absence. This magnifying effect makes Affirmative Action, even at a small scale, an effective remedy for racial inequality resulting from historical discrimination.

On the other hand, even Affirmative Action leads to a lower probability of higher education for the white group, it does not necessarily imply lower learning effort for all white students. The policy effect depends critically on the change in the degree of within-group competition, which, according to (10), can be measured by the probability density at the cut-off point $g(\widehat{\varepsilon})$.

Assumption 4a. The PDF $g(\varepsilon)$ is continuous and unimodal, i.e., there exists $\widetilde{\varepsilon} \in [\underline{\varepsilon},\ \overline{\varepsilon}]$ such that $g(\widetilde{\varepsilon})$ is the highest; $g(\varepsilon)$ is increasing for $\varepsilon \in [\underline{\varepsilon},\ \widetilde{\varepsilon})$ and decreasing for $\varepsilon \in (\widetilde{\varepsilon},\ \overline{\varepsilon}]$.

Proposition 3. *When Affirmative Action lowers the degree of competition among white students, namely* $g(G^{-1}(1 - \frac{\Sigma - \lambda \Sigma^B}{1-\lambda})) < g(G^{-1}(1 - \frac{\Sigma}{1-\lambda}))$*, it leads to lower learning effort and lower human capital achievement for all white students; when Affirmative Action sufficiently increases the degree of competition, namely* $g(G^{-1}(1 - \frac{\Sigma - \lambda \Sigma^B}{1-\lambda})) \gg g(G^{-1}(1 - \frac{\Sigma}{1-\lambda}))$*, it leads to sufficiently higher learning effort in basic education and higher human capital achievement for all white students; in between the results are mixed.*

Proof. See the Appendix. ■

This proposition clearly illustrates how within-group competition plays an essential role in determining the policy impact of Affirmative Action on human capital accumulation for white students. When the total capacity of higher education is very limited, namely $G^{-1}(1 - \frac{\Sigma}{1-\lambda}) > \widetilde{\varepsilon}$, introducing Affirmative Action would further cut the higher education opportunity for the white group. As it is now too hard to get, white students just give up and choose lower effort. Then efficiency trade-off is unavoidable in human capital accumulation between the two groups. On the other hand, if the total capacity of higher education is not too limited, namely $G^{-1}(1 - \frac{\Sigma}{1-\lambda}) < \widetilde{\varepsilon}$, introducing Affirmative Action would add some competition pressure. As it is initially too easy to get but now somewhat more difficult, white students choose more effort in the attempt of securing their opportunities. Depending on the shape of the probability density function $g(\varepsilon)$, if the increase in the within-group competition effect is strong enough, it may offset the negative impact from lower group-wide probability of higher education. In that case, Affirmative Action improves human capital accumulation for the entire population, and efficiency trade-off between the two groups need not arise. This possibility is illustrated by the numerical examples below.

Numerical example 1. Parameters: $H_{-1}^{B} = 2.5$, $H_{-1}^{W} = 3$, $\lambda = 0.2$; $A = 1.4$, $B = 0.8$, $\theta_1 = 1$, $\theta_2 = 1$; $\Sigma = 0.79$; and $g(\varepsilon)$ is a symmetric triangle on $[-0.2,\ 0.2]$.

With these parameters, the initial human capital inequality between the two groups is 20%, namely the human capital endowment of the white group is 20% higher than that of the black group for the parent generation. If there were only basic education in this economy, since white students investment more effort than black students according to (5), the resulting human capital inequality would be 28%.[9] Yet with the higher education opportunity available only to the white group, white students are induced to invest even more effort according to (11), and the resulting human capital inequality becomes 56%, twice as high as that without the education hierarchy. This illustrates the result given in Proposition 1, namely without Affirmative Action, limited access to higher education that is based purely on test score ranking is actually discriminatory against the historically discriminated, and exacerbates the racial inequality.

The equilibrium outcomes under Affirmative Action policy intervention is reported in Figure 1. In graph 1 and 2, the competition effect can be clearly seen when the policy allots an increasing number of college seats from the white group to the black group. For white students, the increase in within-group competition

[9] The lack of convergence in the human capital levels of the two groups overtime is an artifact of the lack of concavity in the education production technology and does not change the main results qualitatively.

dominates the decrease in group-wide probability, so they monotonically increase their effort in basic education. For black students, within-group competition increases first and then starts to decrease, just as the probability density function $g(\varepsilon)$, and so does their effort in basic education. In graph 3 and 4, the human capital achievement for black students keeps increasing, and that for white students increases first and then starts to decrease. So small scale of Affirmative Action has positive impact on human capital accumulation for both groups, thanks to the presence of within-group competition. In graph 5 and 6, the economy-wide aggregate human capital and human capital inequality are reported. It can be seen that small scales of Affirmative Action improve both aggregate efficiency and equality.

Partly as a sensitivity test, numerical example 2 considers the case when higher education technology is less productive, i.e., $B = 0.45$. The results are reported in Figure 2. Here students value leisure more than learning at the higher education stage, which in turn affects their effort at the basic education stage. For the white group, the increase in within-group competition strongly dominates the decrease in the group-wide probability, so both the learning effort and human capital achievement increase with the Affirmative Action policy, mirroring that for the black group. Affirmative Action has positive impact on aggregate efficiency and equality, at both small and big scales.

In these examples, the critical condition is that the capacity of higher education is not too limited. So without policy intervention, higher education is almost a sure thing among white students, and there is little competition. Then Affirmative Action introduces sufficiently strong within-group competition, and lead to the desirable positive impact on human capital accumulation for both groups.

5. Affirmative Action II: test score adjustment

In this section we consider an alternative form of Affirmative Action: certain points s^B are automatically added to black students' test scores before ranking them with the white students.[10] The analysis also serves as a robustness check on the policy impact of Affirmative Action in various forms. Unlike the previous form where between-group competition is excluded by policy design, the current form does allow between-group competition. A student can increase his own probability of getting higher education admission by outperforming not only students in his own group, but also those in the other group. Hence the group-wide probability Σ^B and Σ^W is not exogenously given, but arises as the equilibrium outcome from competition.

For technical reasons, hereinafter we adopt an alternative assumption in place of Assumption 3a.

Assumption 3b. $\varepsilon \tilde{} \; dist(-\infty, \; +\infty)$

With this assumption, there is never perfect stratification between the two groups. No matter how big is the initial racial inequality in human capital endowment, there is always a small yet positive probability that a sufficiently big random shock shuffles the test score ranking. So even without policy intervention, some black students will get higher education admission. The presence of between-group competition even without policy intervention serves as a more suitable benchmark for comparison, and also is a very convenient technical feature that guarantees the existence of an equilibrium. This will become clear in the coming equation (17).

For arbitrary test score adjustment $s^B \geq 0$ ($s^B = 0$ means no Affirmative Action), there is a distribution of after-adjustment test scores for the entire popu-

[10] Or equivalently a lower entrance threshold is imposed for black students relative to that for white students.

lation, which depends on both the optimal effort for the two groups and the scale of the policy. For this distribution, there exists a cut-off test score $\widehat{s}$, such that all students with scores above $\widehat{s}$ get higher education admission. Consequently, the group-wide probability of attending higher education can be determined by

$$\Sigma^B = \Pr(H^{B,1} + s^B + \varepsilon > \widehat{s}) \tag{12}$$

$$\Sigma^W = \Pr(H^{W,1} + \varepsilon > \widehat{s}) \tag{13}$$

In equilibrium, the cut-off test score $\widehat{s}$ is such that the capacity constraint in higher education is satisfied:

$$\lambda \Pr(H^{B,1} + s^B + \varepsilon > \widehat{s}) + (1 - \lambda) \Pr(H^{W,1} + \varepsilon > \widehat{s}) = \Sigma \tag{14}$$

Taking the cut-off test score $\widehat{s}$ as given, individual perceived probability of getting higher education admission is

$$\sigma^B = \Pr(h^{B,1} + s^B + \varepsilon > \widehat{s}) \tag{15}$$

$$\sigma^W = \Pr(h^{W,1} + \varepsilon > \widehat{\varepsilon}) \tag{16}$$

Since a student competes with all other students in the population for the higher education opportunity, the marginal effect of increasing learning effort in basic education on higher education admission is the same for both groups:

$$\frac{\partial \sigma^r}{\partial e^{r,1}} = (\lambda g(\widehat{\varepsilon}^B) + (1 - \lambda) g(\widehat{\varepsilon}^W)) A H^r_{-1} \tag{17}$$

where $\widehat{\varepsilon}^B = G^{-1}(1 - \Sigma^B)$ and $\widehat{\varepsilon}^W = G^{-1}(1 - \Sigma^W)$ in the equilibrium.[11]

[11] This characterization of the competition effect in (17) is only valid under Assumption 3b. Otherwise, under Assumption 3a with $\varepsilon \sim dist[\underline{\varepsilon}, \overline{\varepsilon}]$, $\frac{\partial \sigma^B}{\partial e^{B,1}} = 0$ when $\widehat{\varepsilon}^B < \underline{\varepsilon}$ or $\widehat{\varepsilon}^B > \overline{\varepsilon}$, yet $\frac{\partial \sigma^B}{\partial e^{B,1}} = (\lambda g(\widehat{\varepsilon}^B) + (1 - \lambda) g(\widehat{\varepsilon}^W)) A H^B_{-1}$ when $\widehat{\varepsilon}^B = \underline{\varepsilon}$ or $\overline{\varepsilon}$. Even if $g(\underline{\varepsilon}) = g(\overline{\varepsilon}) = 0$ under Assumption 4a, since $g(\widehat{\varepsilon}^W) > 0$ for generic $\widehat{\varepsilon}^W$, there is discontinuity in the competition effect term for the black group. Parallel argument also shows that there is discontinuity in the competition effect term for the white group. Assumption 3b is hence necessary for the Brouwer's fixed point theorem to apply so that an equilibrium exists.

Proposition 4. *Given the test score adjustment for black students s^B, an equilibrium exists.*

Proof. See the Appendix. ■

Given an Affirmative Action policy s^B, this proposition guarantees the existence but not the uniqueness of the equilibrium. Moreover, since the system of equations governing the equilibrium is highly non-linear, no further analytical results can be obtained. However, it is intuitive to expect policy effects analogous to the previous form of Affirmative Action. If Affirmative Action does not sufficiently increase the degree of competition, efficiency trade-off seems unavoidable in human capital accumulation between the two groups. Yet if Affirmative Action does sufficiently increase the degree of competition, then both groups may choose more effort and have higher human capital achievements. The following assumption simply parallels that of Assumption 4a and serves as a natural extension to measure the degree of competition over the infinite support.

Assumption 4b. The PDF $g(\varepsilon)$ is continuous and unimodal, i.e., there exists $\widetilde{\varepsilon} \in (-\infty, \ +\infty)$ such that $g(\widetilde{\varepsilon})$ is the highest; $g(\varepsilon)$ is increasing for $\varepsilon \in (-\infty, \ \widetilde{\varepsilon})$ and decreasing for $\varepsilon \in (\widetilde{\varepsilon}, \ +\infty)$.

Again two numerical examples are provided to illustrate the policy effects. All other parameters are taken from the previous examples. Now the distribution of ε, which has positive density on the entire real line, is assumed to be ε ˜ *normal* $(0, \ 0.08)$, where the standard deviation is chosen so that the normal density closely resembles the triangle density used in the previous examples. Also the standard deviation is small relative to the initial inequality in human capital endowment, so the group-wide probability is essentially zero for the black group without policy intervention.

Reported in Figure 3 is numerical example 3 with $B = 0.8$, a counterpart to numerical example 1. It is easy to see that the main pattern remains the same while minor details differ. Small scale of Affirmative Action strongly increases the degree of competition, induces both groups to invest more effort in basic education, and leads to higher human capital achievements for both groups. It has positive impact on both aggregate efficiency and equality in this economy. Compared to college seats allotment where the two groups may face different degrees of competition, test score adjustment implies the same degree of competition for both groups, so the optimal effort levels for the two groups are somewhat parallel.

Reported in Figure 4 is numerical example 4 with $B = 0.45$, a counterpart to numerical example 2. Again this specific form of Affirmative Action changes the policy effects only quantitatively but not qualitatively. Since students value leisure more than learning at the higher education stage, here even reasonably big scale of Affirmative Action can have positive impact on improving student effort and human capital achievement for both groups. Two minor differences are worth noting. First, unlike the college seats allotment where aggregate efficiency is strictly increasing with Affirmative Action, here aggregate efficiency eventually drops when the test score adjustment becomes too large. Second, unlike all three previous examples where human capital equality strictly improves with Affirmative Action, here racial equality actually worsens in a small range before it starts to improve. Nevertheless, the main results are robust to both education technology parameters and various forms of Affirmative Action. Overall when competition for higher education is initially low among students, Affirmative Action can introduce more competition and lead to higher human capital achievements for both groups. There need not be efficiency trade-off in human capital accumulation between the two groups or efficiency loss in enhancing racial equality.

6. Discussion and conclusion

This paper studies the policy impact of Affirmative Action in hierarchical education, explicitly taking student competition for higher education opportunity into account. It is shown that without policy intervention, limited access to higher education that is based purely on test score ranking is actually discriminatory against the historically discriminated, and exacerbates racial inequality. Affirmative Action improves racial equality and need not invoke any efficiency loss in human capital accumulation for either group. When the capacity for higher education is not too limited, there is little competition among white students initially, Affirmative Action can introduce more competition and lead to more learning effort and higher human capital achievement for all students in both groups. Affirmative Action thus has positive impact on both aggregate efficiency and social equality.

For tractability, this model assumes homogeneous individuals within each group, so ex ante all black students share the same probability to benefit from Affirmative Action, and all white students face the same probability to pay the cost for Affirmative Action. This assumption allows us to focus on the role of within-group competition, yet without heterogeneous individuals within each racial group, we cannot study the differential incidence of Affirmative Action across individuals. As shown in Chan and Eyster (2002), under simple form of Affirmative Action, the lower fraction of the white group bears most of the cost, and the upper fraction of the black group reaps most of the benefit. In the current model, if we further divide each racial group into multiple sub-groups differing in their human capital endowment, it is intuitive to expect that the same mechanism remains valid under finer tuned Affirmative Action, namely instead of choosing Σ^B and Σ^W, the policy can specify $\Sigma^{B,i}$ for all the i black sub-groups, and $\Sigma^{W,j}$ for all the j white sub-groups respectively. Model extension in this direction may generate interesting results with regard to different sub-groups' policy preferences and hence political support for Affirmative Action, and is left for future research.

In this model, there is no fundamental difference between racial groups. They differ only by the initial human capital endowment. Consequently, what we call racial inequality here can also be interpreted as income inequality. Affirmative Action aimed at improving racial equality can also be viewed as "the War on Poverty." Since the two-group model is readily extended to multiple groups, it direct implication is that the door of higher education should never be completely shut to those individuals with inferior socioeconomic background, as long as higher education opportunity is also valuable to them. By redistributing some probability of higher education from top groups with superior socioeconomic background to bottom groups with inferior socioeconomic background, Affirmative Action can introduce more competition at both the top and the bottom, and hence have positive impact on aggregate efficiency and social equality.

While Affirmative Action faces many challenges within the United States recently, it is being emphasized in other countries. Two examples are Britain and India (The Economist, 2004). The British government recently issued tough new benchmarks for the number of state-school pupils that universities ought to take, who are more likely to be from poor background. And the India government has been setting quota to secure jobs and education opportunities for the poorer half of the population, including various tribal groups and low castes in the Hindu caste system. This paper suggests that identity-conscious (either race or socioeconomic background) Affirmative Action, especially at small scale, may have positive impact on both aggregate efficiency and social equality. As a complement, other studies show that switching from identity-conscious to identity-blind policy often incurs sizeable efficiency loss.[12] Hence our current policy debate on whether and how to apply Affirmative Action will have significant impact on the efficiency and equality of the US economy.

[12] For example, see Fryer, Loury and Yuret (2003) and Epple, Romano and Sieg (2003).

References

Arrow, Kenneth J. "The Theory of Discrimination," in Orley Ashenfelter and Albert Rees, eds., Discrimination in Labor Markets, Princeton, NJ: Princeton Press, 1973, pp. 3-33.

Chan, Jimmy and Eyster, Erik. "Does Banning Affirmative Action Lower College Student Quality?" American Economic Review, June 2003, 93(3), pp. 858-72.

——— and ———. "Admission Impossible? Self Interest and Affirmative Action." Johns Hopkins University, working paper #479, July 2002.

**Chung, Kim-Sau. "Role Models and Arguments for Affirmative Action." American Economic Review, June 2000, 90(3), pp. 640-48.

———. "Affirmative Action as an Implementation Problem." University of Wisconsin mimeo, 1999.

Coate, Stephen and Loury, Glenn C. "Will Affirmative Action Policies Eliminate Negative Stereotypes?" American Economic Review, December 1993, 83(5), pp. 1220-40.

Currie, Janet and Thomas, Duncan. "Race, Children's Cognitive Achievement and the Bell Curve," NBER working paper w5240, 1995.

De Fraja, Gianni. "Reverse Discrimination and Efficiency in Education." International Economic Review, forthcoming.

Epple, Dennis; Romano, Richard and Sieg, Holger. "The Practice and Proscription of Affirmative Action in Higher Education: an Equilibrium Analysis." NBER working paper w9799, 2003.

Fryer, Roland G. Jr., Loury, Glenn C. and Yuret, Tolga. "Color-Blind Affirmative Action," NBER working paper w10103, 2003.

Holzer, Harry and Neumark, David. "Assessing Affirmative Action," Journal of Economic Literature, September 2000, 38(3), pp. 483-568.

Neal, Derek and Johnson, William. "The Role of Premarket Factors in Black-White Wage Differences," Journal of Political Economy, 1996, 104(5), pp. 869-95.

Phelps, Edmund S., "The Statistical Theory of Racism and Sexism," American Economic Review, September 1972, 62(4), pp. 659-61.

Rogers, William M. and Spriggs, William E. "What Does the AFQT really measure: Race, Wages, Schooling and the AFQT score," Review of Black Political Economy, 1996, 24(4), pp. 13-46.

Schotter, Andrew and Weigelt, Keith. "Asymmetric Tournaments, Equal Opportunity Laws, and Affirmative Action: Some Experimental Results," Quarterly Journal of Economics, May 1992, 107(2), pp. 511-39.

Steele, Shelby. "The Content of Our Character," New York: St Martin's Press, 1990.

Su, Xuejuan. "The Allocation of public Funds in a Hierarchical Educational System," Journal of Economic Dynamics and Control, December 2004, 28(12), pp. 2485-2510.

———. "A Political Economy Model of Public Budget Allocation across Education Stages," 2004, University of Alabama, mimeo.

The Economist, October 9th-15th issue, 2004, pp. 53; November 13th-19th issue, 2004, pp. 47.

Appendix.

Proposition 1.

Proof. The initial racial inequality is given by $\frac{H_{-1}^W}{H_{-1}^B}$. After the two stages of education, the resulting racial inequality is $\frac{H^W}{H^B} > \frac{H^{W,1}}{H^{B,1}} > \frac{H_{-1}^W}{H_{-1}^B}$, where the first inequality follows when a fraction Σ^W of white students enjoy higher education but no black students do, and the second inequality follows because white students invest more learning efforts in basic education than black students. ■

Proposition 2.

Proof. Without Affirmative Action and $\Sigma^B = 0$, the optimal learning effort of a black student in basic education is $e^{B,1} = \underline{e}^{B,1}$. With Affirmative Action and $\Sigma^B > 0$, the optimal learning effort is given by the counterpart of (11) with the superscript $r = B$ replacing $r = W$. While the marginal cost curve remains the same, Affirmative Action shifts the marginal benefit curve upward due to the two additional components associated with higher education. It follows that Affirmative Action leads to higher learning effort in basic education for all black students, and higher learning effort for a fraction Σ^B of black students in higher education, and hence higher human capital achievement for all black students. ■

Proposition 3.

Proof. From (10) and (11), it is easy to see that when $g(G^{-1}(1 - \frac{\Sigma - \lambda\Sigma^B}{1-\lambda})) < g(G^{-1}(1 - \frac{\Sigma}{1-\lambda}))$, the marginal benefit curve for white students shifts downward while the marginal cost curve remains the same, which leads to lower learning effort and lower human capital output from basic education for all white students. From (6), the learning effort in higher education is also lower for the fraction of $\Sigma^W = \frac{\Sigma - \lambda\Sigma^B}{1-\lambda}$ who gets higher education admission, and it lowers down to zero for the fraction $\frac{\lambda\Sigma^B}{1-\lambda}$ who would otherwise get higher education admission. Overall Affirmative Action leads to lower learning effort and lower human capital achievement for all white students.

When $g(G^{-1}(1-\frac{\Sigma-\lambda\Sigma^B}{1-\lambda})) \gg g(G^{-1}(1-\frac{\Sigma}{1-\lambda}))$, the marginal benefit curve for white students shifts downward due to the lower probability of getting higher education admission, but it shifts upward due to the sharp increase in the within-group competition. The upward shift dominates the downward shift, which leads to sufficiently higher learning effort and higher human capital output from basic education for all white students. With regard to higher education, the fraction $\Sigma^W = \frac{\Sigma-\lambda\Sigma^B}{1-\lambda}$ who gets admission invests more effort and hence accumulates more human capital in higher education; the fraction $\frac{\lambda\Sigma^B}{1-\lambda}$ loses the opportunity, but the loss in human capital output from higher education is more than offset by the gain from basic education; and the remaining fraction $1-\frac{\Sigma}{1-\lambda}$ remains out of higher education. Overall Affirmative Action leads to sufficiently higher learning effort in basic education and higher human capital achievement for all white students.

In between, if the upward shift of the marginal benefit curve due to the increase in within-group competition does not offset the downward shift due to the lower group-wide probability, then it is equivalent to the first case, and Affirmative Action leads to lower learning effort and lower human capital achievement for all white students. On the other hand, if the upward shift dominates the downward shift but not at a sufficiently high level, it leads to higher learning effort and higher human capital output from basic education, but the gain in human capital output from basic education is not sufficient to offset the loss for the fraction $\frac{\lambda\Sigma^B}{1-\lambda}$ who would otherwise get higher education admission. Overall Affirmative Action leads to higher learning effort in basic education for all white students, higher human capital achievement for the fraction $1-\frac{\lambda\Sigma^B}{1-\lambda}$ of white students, but lower human capital achievement for the fraction $\frac{\lambda\Sigma^B}{1-\lambda}$ of white students. ■

Proposition 4.

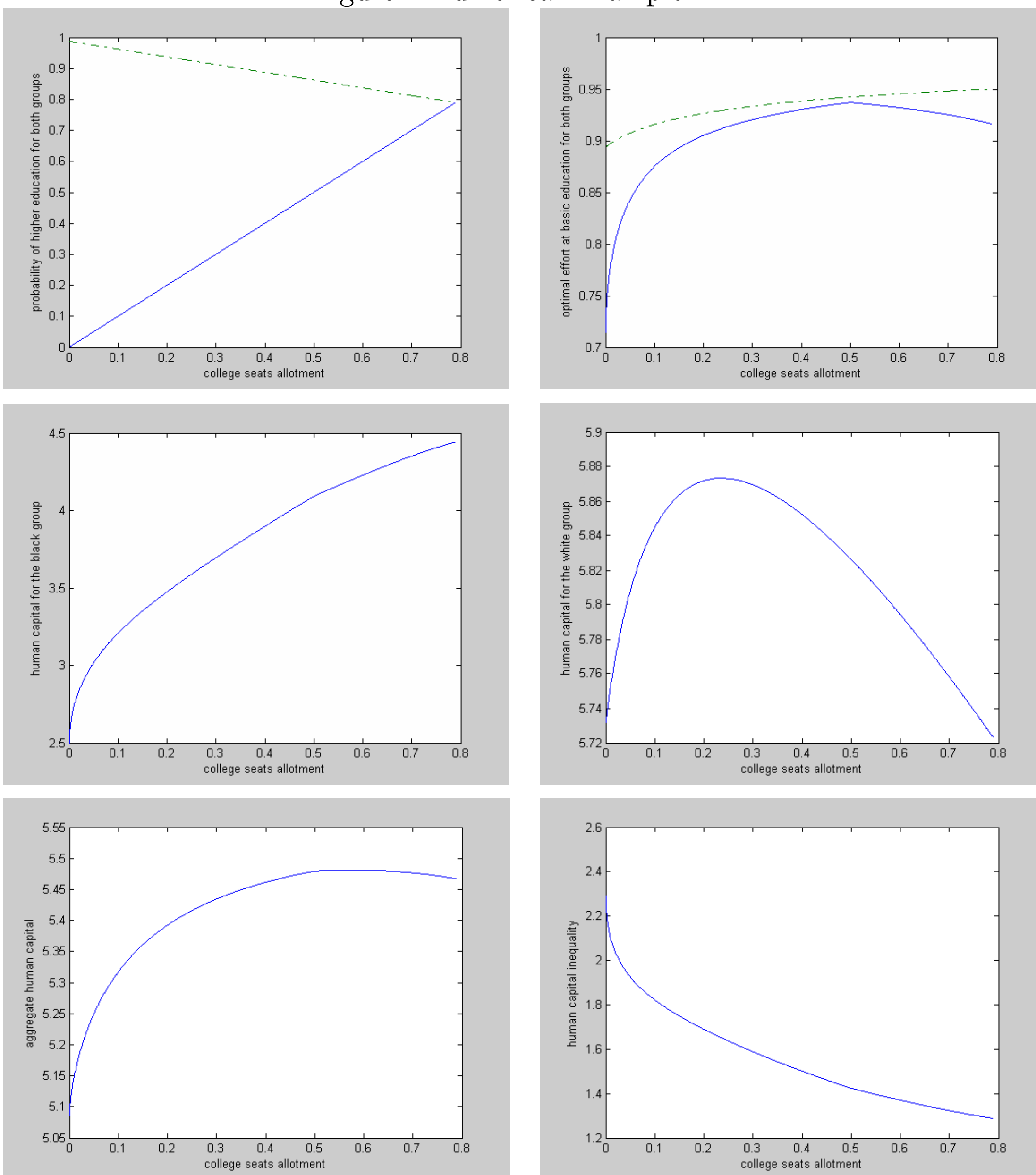
Proof. An equilibrium is a fixed point of the system of three equations with three unknowns: $e^{B,1}$, $e^{W,1}$ and $\widehat{s}$. The three equations are (11) with $\frac{\partial \sigma^W}{\partial e^{W,1}}$ given by (17), it counterpart where $r = B$, and (14) (with variables σ^B and σ^W given by(15) and (16) together with the symmetry condition). Under Assumption 4b, $g(\varepsilon)$ is continuous on $(-\infty,\ +\infty)$, so the system of equations is continuous.

It has been shown that $e^{r,1} \geq \underline{e}^{r,1}$ according to (5) and (11) (when $\sigma^r = 0$ and $\frac{\partial \sigma^r}{\partial e^{r,1}} = 0$), and it is obvious that $e^{r,1} \leq \overline{e}^{r,1}$, with $\overline{e}^{r,1}$ given by the following equation:

$$AH^r_{-1} + ABH^r_{-1}e^{r,2*} + g(\widetilde{\varepsilon})AH^r_{-1}v^{W,2}(e^{W,1}) = \frac{\theta_1}{1 - e^{W,1}}$$

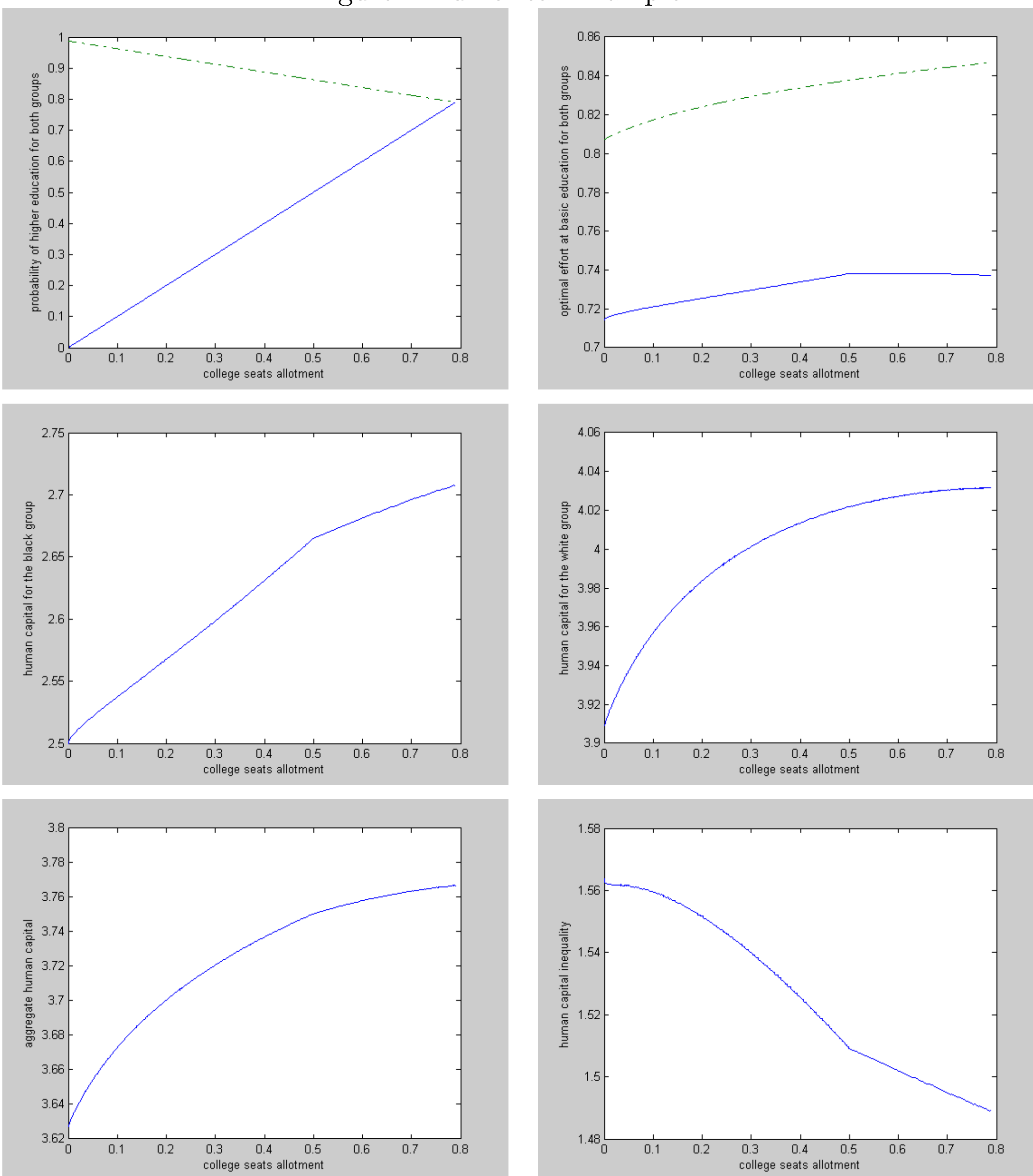
where marginal benefit is the highest when $\sigma^r = 1$ and $\frac{\partial \sigma^r}{\partial e^{r,1}} = g(\widetilde{\varepsilon})AH^r_{-1}$. So $e^{r,1}$ is abounded by $[\underline{e}^{r,1},\ \overline{e}^{r,1}]$. Also $\widehat{s}$ is bounded by $[\min\{AH^B_{-1}\underline{e}^{B,1} + s^B,\ AH^W_{-1}\underline{e}^{W,1}\} + G^{-1}(\Sigma),\ \max\{AH^B_{-1}\overline{e}^{B,1} + s^B,\ AH^W_{-1}\overline{e}^{W,1}\} + G^{-1}(\Sigma)]$. Thus Brouwer's fixed point theorem applies, and an equilibrium exists. ■

Figure 1 Numerical Example 1



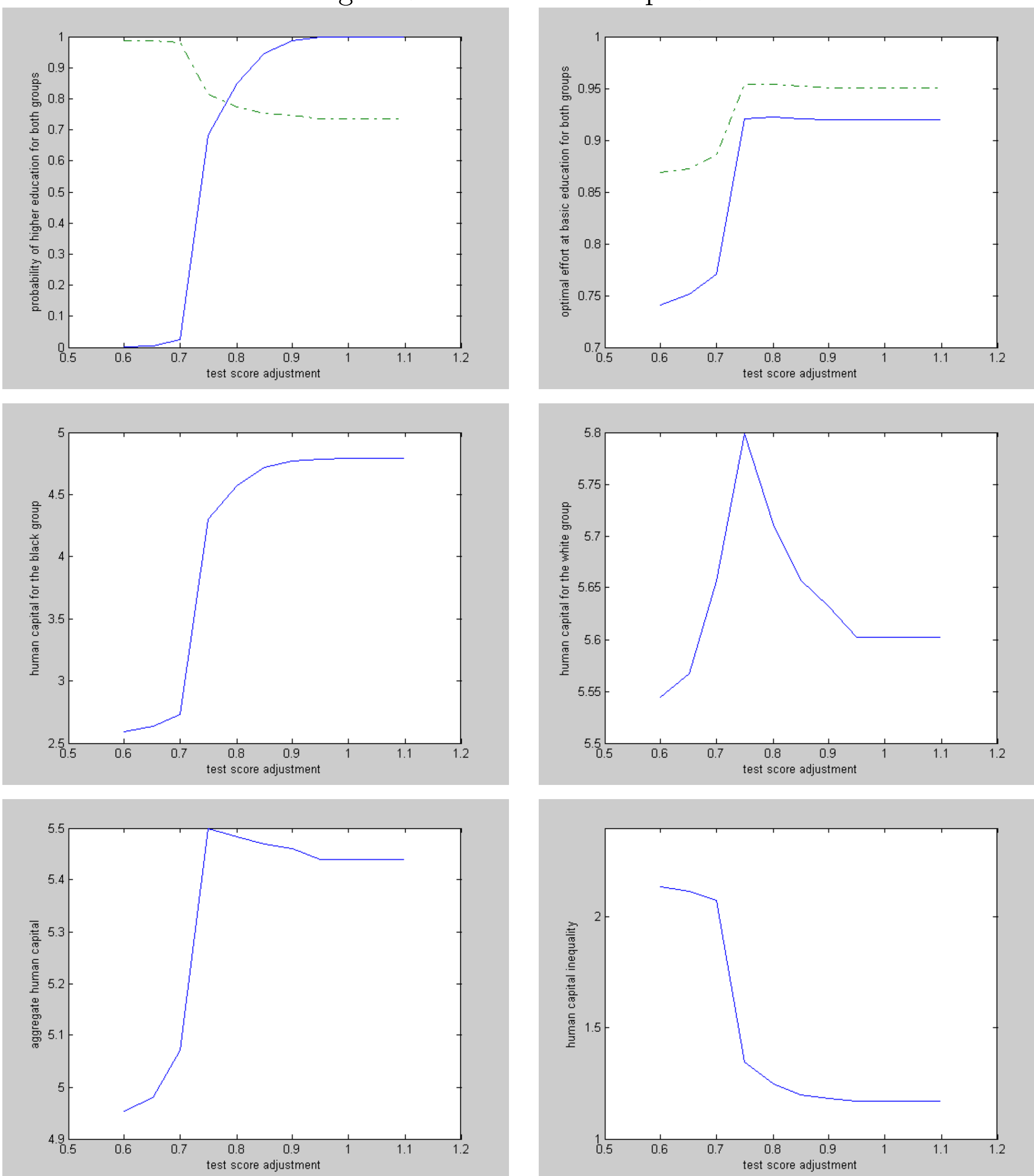
Note: in graph 1 and 2, solid line - the black group, dashed line - the white group.

Figure 2 Numerical Example 2



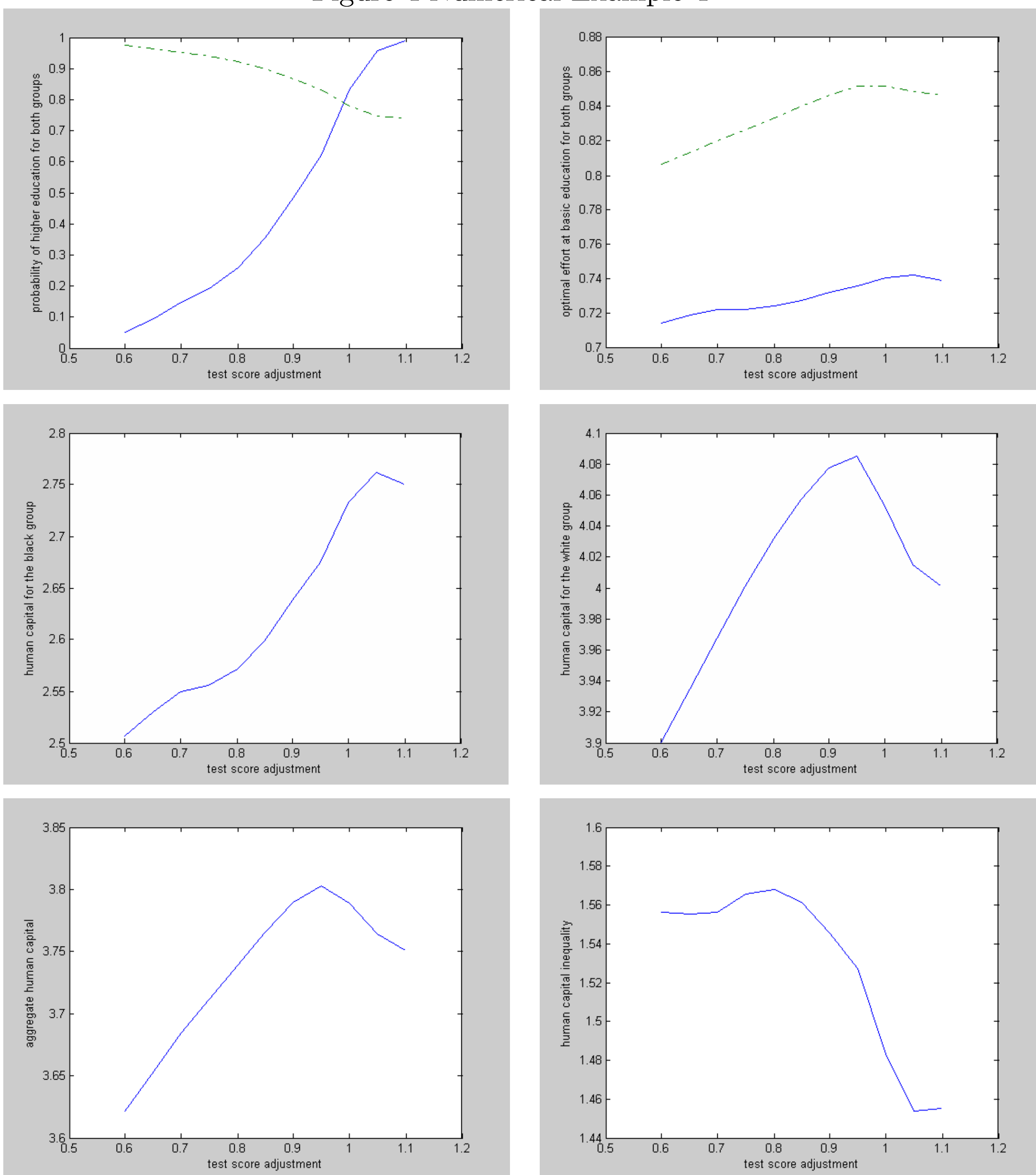
Note: in graph 1 and 2, solid line - the black group, dashed line - the white group.

Figure 3 Numerical Example 3



Note: in graph 1 and 2, solid line - the black group, dashed line - the white group.

Figure 4 Numerical Example 4



Note: in graph 1 and 2, solid line - the black group, dashed line - the white group.