Testing for a Unit Root in the Asymmetric Nonlinear Smooth Transition Framework

Razvan Pascalau*
Department of Economics, Finance and Legal Studies
University of Alabama

August 16, 2007

Abstract

This paper proposes a simple testing procedure to detect the presence of nonlinear but global stationary logistic smooth transition autoregressive processes. This testing procedure nests the one developed by Kapetanios et al. (2003) that accommodates specifically the alternative of a global stationary ESTAR process. The present work makes a threefold contribution to the literature. First, it derives the limiting non-standard distribution of the proposed test. Second, the paper finds via Monte Carlo simulations that under the alternative of a globally stationary LSTAR process, this new test has better power than a standard Dickey-Fuller test. Third, an empirical application of the test to real exchange data in OECD countries rejects the null of a unit root more often than a simple Dickey-Fuller. This finding provides more evidence in favor of nonlinear PPP mean-reversion.

Keywords:
JEL-Classification:

*Razvan Pascalau, Department of Economics, Finance, & Legal Studies, Culverhouse College of Commerce and Business Administration, University of Alabama, 200 Alston Hall, Box 870224, Tuscaloosa, AL 35487, USA; Tel: +1-205-348-7592, Email: rpascala@cba.ua.edu

1 Introduction

The present paper adds to both the literature on unit root testing and modeling time series nonlinearities. Given the poor power performance of the Dickey-Fuller (abbreviated DF) unit root test under the simple linear ARMA framework, researchers have sought to increase power by developing unit root tests that are robust to various nonlinearities present in the data. Given that any neglected nonlinearity of an otherwise stationary series may bias the DF unit root test towards accepting the null, these new tests may display increased power when the alternative is true but nonlinear around a level and/or trend.

Several studies, including but not limited to Maddala (1977) and more recently Lin and Granger (1994) and Teräsvirta (1998) recommend the use of the logistic function to model the smooth transition between a number of regimes of a regime-switching process. The logistic function appears particularly suited to characterize the asymmetric behavior of nonlinear processes like the business cycle for instance. Additionally, empirical applications concern the modeling of real exchange, unemployment, and real interest rates.

For example, the persistent failure of the standard DF test to reject the null of a unit root for bilateral and real effective exchange and real interest rates (as the purchasing power parity (PPP) and the mean reversion of the marginal product of capital theories would imply, respectively) researchers have modified the DF test in several ways. Of direct interest to the current study are the papers of Bec et al. (2004), Leybourne et al. (1998), Sollis et al. (2002), and Kapetanios et al. (2003).

Bec et al. (2004) comes closest to the approach followed in this paper. Specifically, under the alternative, they approximate the global LSTAR stationary process through a second-order Taylor series. Although this ensures a more parsimonious representation, this framework limits the range of possible alternatives against which the test will have power. Moreover, their discussion is somewhat restrictive in that it imposes a unit root in the lower regime.

Leybourne et al. (1998), and Sollis et al. (2002) develop unit root tests that modify the deterministic component of the standard DF test to include a logistic smooth transition (LSTAR) function. By doing so, the authors aim to increase the power of the test through the approximation of the possibly slow and gradual regime change. As Perron (1989) points out, a single structural change in the trend and/or level of an otherwise stationary series is sufficient to render the respective series behave like an I(1) process.

Leybourne et al. (1998) use a special version of the logistic function where the transition function is a linear time trend. Thus, their test yields a model with deterministically changing parameters which can be viewed alternatively as a test for parameter variability. Sollis et al. (2002) uses the one period lag of the demeaned series as the transition variable. Their testing framework requires the LSTAR de-trending of the series, saving the residuals, and then performing an ADF test on these residuals. However, the LSTAR de-trending step involves the use of a nonlinear least squares algorithm which in turn poses convergence issues. Indeed, as Sollis et al. (2002) point out, they experienced difficulties in achieving convergence in the iterative algorithm, especially when the unit root null is true. Moreover, any error from the first step estimation carries over to the next step. Finally, the latter two studies stop short of providing the asymptotic null distribution of their tests.

Given these limitations, this paper follows Teräsvirta (1994) and Kapetanios et al. (2003) to provide a simpler and more straightforward approach. Teräsvirta (1994) provides the key insight. He uses a third-order Taylor series approximation of the LSTAR component to test the null of linearity of a stationary process. This paper builds on this idea and allows the series to be I(1) under the null. The new test proposes the alternative of a nonlinear but globally stationary LSTAR process. In contrast to Bec et al. (2004), the testing framework of this paper expands the range of possible nonlinear alternatives. Additionally, the present paper investigates the small sample behavior of the test both with and without an explosive regime in the lower regime. Kapetanios et al. (2003) provide the ESTAR version of this test (denoted KSS hereafter). However, the test proposed in this paper nests the KSS one. Moreover, it allows both for symmetric and asymmetric adjustment under the alternative.

Therefore, the contribution of this paper is threefold. First, the paper establishes the limiting nonstandard asymptotic distribution of the test. Second, it performs Monte Carlo simulations to highlight the increased power over the standard DF test when the alternative comprises a smooth transition regime. Third, it applies the new test to several real exchange rate data and as expected, it finds more evidence of non-linear mean reversion than previous studies.

2 The model

In general terms, a univariate smooth transition regression (STAR) model of order 1 has the following representation:

$$y_t = \alpha y_{t-1} + \beta y_{t-1} G(\theta; y_{t-d}; c) + \epsilon_t, t = 1, \ldots, T \tag{1}$$

where $\epsilon_t \sim iid(0, \sigma^2)$, and α and β are unknown parameters. The logistic transition function has the following form:

$$G(\theta; y_{t-d}) = [1 + exp(-\theta(y_{t-d} - c))]^{-1}, \tag{2}$$

where we assume that $\theta > 0$, $d \geq 1$ is the delay parameter, and c is the location parameter (i.e. threshold). In the transition function $G(\theta; y_{t-d}; c)$, θ is the slope parameter. The transition function is a bounded function of the transition variable y_{t-d} and continuous everywhere in the parameter space for any value of y_{t-d}. Given that $G(\theta; y_{t-d}; c)$ is odd and monotonically increasing whenever $|y_{t-d} - c|$ is large and $y_{t-d} < c$, y_t is effectively generated by the linear model:

$$y_t = \alpha y_{t-1} + \epsilon_t, t = 1, \ldots, T. \tag{3}$$

If $|y_{t-d} - c|$ is large and $y_{t-d} > c$, y_t is virtually generated by:

$$y_t = (\alpha + \beta) y_{t-1} + \epsilon_t, t = 1, \ldots, T. \tag{4}$$

Thus, denoting relationship (3) the low regime and relationship (4) the high regime, one can note that the parameters α and $\beta G(\theta; y_{t-d}; c)$ change monotonically as a function of y_{t-d} from α to $\alpha + \beta$.

An LSTAR model of order one characterizes asymmetric behavior of processes whose dynamic properties are different in expansions from what they are in recessions for instance, such that the transition from one regime to the other is smooth over time. Using equation (2) in (1) one obtains the logistic STAR (LSTAR) model,

$$y_t = \alpha y_{t-1} + \beta y_{t-1}[1 + exp(-\theta(y_{t-d} - c))]^{-1} + \epsilon_t, \tag{5}$$

which, following Kapetanios et al. (2003) can be reparameterised as

$$\Delta y_t = \delta y_{t-1} + \beta y_{t-1}[1 + exp(-\theta(y_{t-d} - c))]^{-1} + \epsilon_t, \tag{6}$$

where $\delta = \alpha - 1$. When $\theta = 0$, the transition function $G(\theta; y_{t-d}; c) \equiv 1/2$ so that the LSTAR model nests a linear model. Conversely, when $\theta \to \infty$ the LSTAR model approaches a switching regime with two distinct regimes.

As mentioned, Leybourne et al. (1998) use a linear time trend t in (2) instead of the stochastic variable y_{t-d}. Kapetanios et al. (2003) employ the exponential version in 1 where $G(\theta; y_{t-d}) = 1 - exp(-\theta y_{t-d}^2)$.

Chan and Tong (1986) introduced the LSTAR model in the time series literature. Empirical applications include analysis of the PPP hypothesis (Sollis et al. (2002), modeling asymmetric behavior of macroeconomic variables such as industrial production and unemployment rate (see Proietti (2003) for references), money demand (Teräsvirta and Eliasson (2001)), and the list could go on. When testing the stationarity of real exchange rates for instance, one must have $\beta < 0$ and $\delta + \beta < 0$ for the respective series to be globally stationary. Thus, the larger the deviation from long run PPP, the stronger the tendency to revert to equilibrium.

The null hypothesis where $\delta = \beta = \theta = 0$ gives a special case of a linear unit root. Under the alternative, even if $\delta = 0$ but $\theta > 0$, y_t follows a nonlinear but globally stationary process provided that $-2 < \beta < 0$. In practice, the value of the delay factor should be chosen via a goodness of fit maximization procedure over $d = 1, \ldots, d_{max}$. However, a value of $d = 1$ is most often used in the applied literature (see Teräsvirta (2006)) and this view is adopted here as well.

Imposing $d = 1$ gives one possible LSTAR testing framework:

$$\Delta y_t = \delta y_{t-1} + \beta y_{t-1}[1 + exp(-\theta(y_{t-1} - c))]^{-1} + \epsilon_t. \tag{7}$$

Under the null $H(0) : \delta = \theta = \beta = 0$ whereas under the alternative $H(1)$: $\delta + \beta < 0$ and $\theta > 0$. One might expect that a standard ADF test may lose power when the alternative is stationary but nonlinear. However, this testing framework falls under the criticism of Davies (1987) due to the unidentified parameters β and θ under the null, respectively (i.e., when $\beta = 0$, then θ can be anything, while when θ=0, then $\beta = 0$ cannot be distinguished from δ).

Additionally, following the practice in the literature (e.g., Balke and Fomby (1997) for threshold autoregressive models and Kapetanios et al. (2003) for ESTAR models) one can impose $\delta = 0$. This assumption translates into a unit root for the low regime, but stable dynamics everywhere else (i.e., for large values of y_{t-1}). The argument of Keynes (1936) (pp. 314) that contractions in the economy are more violent (but also last for shorter periods) than recessions also supports this assumption. Under the null we have that $H(0) : \beta = \theta = 0$, while under the alternative $H(1)$: $\beta < 0$ and $\theta > 0$. Thus, a second testing framework writes as:

$$\Delta y_t = \beta y_{t-1}[1 + exp(-\theta(y_{t-1} - c))]^{-1} + \epsilon_t. \tag{8}$$

To overcome Davies (1987)'s criticism, I follow Teräsvirta (1994) and use a third-order Taylor series approximation of $G(\theta; y_{t-1}; c)$ with respect to h_{t-1} evaluated at $h_{t-1} = 0$

(where $h_{t-1} = \theta(y_{t-1} - c)$). One gets the following auxiliary regression:

$$\Delta y_t = \gamma_1 y_{t-1}^2 + \gamma_2 y_{t-1}^3 + \gamma_3 y_{t-1}^4 + \epsilon_t. \tag{9}$$

Under the null $H(0) : \gamma_1 = \gamma_2 = \gamma_3 = 0$ and this can be evaluated by means of an F-test. Under the alternative one has that $H(1) : \gamma_1 + \gamma_2 + \gamma_3 < 0$. Thus, the auxiliary regression tests the significance of the score vector from the quasi-likelihood function of the LSTAR model.

Unlike the result in Teräsvirta (1994) of testing linearity against nonlinearity for a stationary process, the F-test in this paper does not have an asymptotic standard normal distribution.

Theorem 2.1 *Under the null of a unit root, the F-test for $\gamma_1 = \gamma_2 = \gamma_3 = 0$ denoted F_{NL} has the following asymptotic distribution:*

$F_{NL} \rightarrow_L \mathbf{v}'\mathbf{Q}^{-1}\mathbf{v}$ *with,*

$$\mathbf{v} = \begin{bmatrix} \frac{1}{3}W(1)^3 - \int_0^1 W(r)dr \\ \frac{1}{4}W(1)^4 - \frac{3}{2}\int_0^1 W(r)^2 dr \\ \frac{1}{5}W(1)^5 - 2\int_0^1 W(r)^3 dr \end{bmatrix}$$

and

$$\mathbf{Q} = \begin{bmatrix} \int_0^1 W(r)^4 dr & \int_0^1 W(r)^5 dr & \int_0^1 W(r)^6 dr \\ \int_0^1 W(r)^5 dr & \int_0^1 W(r)^6 dr & \int_0^1 W(r)^7 dr \\ \int_0^1 W(r)^6 dr & \int_0^1 W(r)^7 dr & \int_0^1 W(r)^8 dr \end{bmatrix}$$

where $W(r)$ is the standard Brownian motion defined on $r \in [0,1]$. Under the alternative hypothesis with the LSTAR model in (8), the F_{NL} statistic is consistent.

Proof 5.1. See the appendix.

This result shows that the test above is free from any nuisance parameters. However, one must note this testing framework has power against both LSTAR and ESTAR type stationarity. Thus, even if the null of a unit root is rejected, one cannot distinguish the exact type of nonlinearity. However, if the series appears global stationary with smooth transition then one can use the approach of Teräsvirta (1994) to distinguish between the two.

There are prior theoretical arguments for restricting the threshold parameter, c, to be zero in many economic and financial applications in the LSTAR function, in which case one obtains the following restricted regression:

$$\Delta y_t = \gamma_1 y_{t-1}^2 + \gamma_3 y_{t-1}^4 + \epsilon_t. \tag{10}$$

Also, note that this restricted regression supports specifically the alternative of an LSTAR-type stationarity. The corresponding F-test has the following limit distribution:

Theorem 2.2 *Under the null of a unit root, the F-test for $\gamma_1 = \gamma_3 = 0$ denoted $\bar{F}_{NL}$ has the following asymptotic distribution:*

$\bar{F}_{NL} \rightarrow_L \bar{\mathbf{v}}'\bar{\mathbf{Q}}^{-1}\bar{\mathbf{v}}$ *with,*

$$\bar{\mathbf{v}} = \begin{bmatrix} \frac{1}{3}W(1)^3 - \int_0^1 W(r)dr \\ \frac{1}{5}W(1)^5 - 2\int_0^1 W(r)^3 dr \end{bmatrix}$$

and

$$\bar{\mathbf{Q}} = \begin{bmatrix} \int_0^1 W(r)^4 dr & \int_0^1 W(r)^6 dr \\ \int_0^1 W(r)^6 dr & \int_0^1 W(r)^8 dr \end{bmatrix}$$

where $W(r)$ is the standard Brownian motion defined on $r \in$ [0,1]. Under the alternative hypothesis with the LSTAR model in (10), the $\bar{F}_{NL}$ statistic is consistent.

Proof 5.2. See the appendix.

So far the y_t process has been assumed to have a mean of zero. However, the result above can be extended easily to allow both for non-zero mean and/or for linear deterministic trends. When y_t contains an intercept, then one needs to demean the data, i.e. $y_t = x_t$ - $\bar{x}$ (where $\bar{x}$ is the sample mean of $x_t = \mu + y_t$), while when y_t includes additionally a deterministic time trend, then one needs to de-trend the data, i.e. $y_t = x_t$ - $\hat{\mu}$ - $\hat{\phi}$ t, where $\hat{\mu}$ and $\hat{\phi}$ are the OLS estimates from $x_t = y_t + \mu + \phi t$. In these cases the result of Theorem 2.1 only needs to be modified to replace $W(r)$ with the demeaned and de-trended standard Brownian motions, respectively.

As Kapetanios et al. (2003) argue, the modeling of intercepts and trends in nonlinear models is not straightforward. As is the case here, the use of the demeaned and/or de-trended data implies a particular view of how to incorporate a level and/or a trend under the alternative. Similarly to the arguments of Kapetanios et al. (2003), although this approach may affect somewhat the finite sample power, the suggested testing procedure is asymptotically similar with respect to intercepts or time trends.

Table 1 shows the tabulated asymptotic critical values of the F_{NL} test for the three cases discussed above. Results have been obtained using 50,000 Monte Carlo simulations, where $T = 1000$.

[Insert Table 1 about here]

Next, one may also want to allow the errors in (6) to display serial dependence. Thus, we can relax the *iid* assumption and consider that the innovations are generated instead by the process $u_t = C(L)\ \epsilon_t$ where $C(L) = \sum_{j=0}^{\infty} c_j L^j$, $\sum_{j=0}^{\infty} |j|^{1/2}|c_j| < \infty$, and as before $\epsilon_t \sim iid(0, \sigma^2)$. The well-established results of Dickey and Fuller (1979) suggest one possible way to correct for serial dependence. Thus, one can extend model (8) to include lags of Δy_t until the residuals behave like white-noise processes:

$$\Delta y_t = \beta y_{t-1}[1 + exp(-\theta y_{t-1})]^{-1} + \sum_{j=1}^{p} \rho_j \Delta y_{t-j} + \epsilon_t \tag{11}$$

Following the approach outlined above, the auxiliary regression employed for testing the null of a unit root now becomes:

$$\Delta y_t = \gamma_1 y_{t-1}^2 + \gamma_2 y_{t-1}^3 + \gamma_3 y_{t-1}^4 + \sum_{j=1}^{p} \rho_j \Delta y_{t-j} + \epsilon_t. \tag{12}$$

Theorem 2.3 *Consider the nonlinear ADF regression in (11). Under the null, the F_{NL} statistic from (12) has the same asymptotic distribution as the one obtained under the assumption of serially independent errors. Under the alternative hypothesis, the F_{NL} statistic is consistent.*

Proof 5.3. See the appendix.

Similarly, the augmented restricted auxiliary regression writes as:

$$\Delta y_t = \gamma_1 y_{t-1}^2 + \gamma_3 y_{t-1}^4 + \sum_{j=1}^{p} \rho_j \Delta y_{t-j} + \epsilon_t. \tag{13}$$

We have the following theorem:

Theorem 2.4 *Consider the nonlinear ADF regression in (13). Under the null, the $\bar{F}_{NL}$ has the same asymptotic distribution as the one obtained under the assumption of serially independent errors. Under the alternative hypothesis, the $\bar{F}_{NL}$ statistic is consistent.*

Proof 5.4. See the appendix.

In practice, standard lag selection criteria like the BIC and/or AIC can be employed to find the optimum lag length. This approach appears adequate given that under the null the model is linear. Alternatively, one can adopt the semi-parametric correction method proposed by Phillips and Perron (1988).

3 Small Sample Properties

This section performs a small-scale Monte Carlo investigation of the small sample size and power performance of the F_{NL} test and compares it with that of KSS, Enders-Granger (EG), and Dickey-Fuller unit root tests, respectively. As mentioned above, when the slope parameter $\theta \to \infty$, the logistic function becomes a threshold model with two distinct regimes:

$$\Delta y_t = \begin{cases} \rho_1 y_{t-1} + \epsilon_t, & \text{if } y_{t-1} \geq \text{c} ; \\ \rho_2 y_{t-1} + \epsilon_t, & \text{if } y_{t-1} < \text{c}. \end{cases} \tag{14}$$

The EG unit root test tests for $\rho_1 = \rho_2 = 0$, where the deterministic component can include both an intercept and/or time trend. Thus, one would expect that for large values of θ, the new F_{NL} and the EG test will perform equally well.

Additionally, the new F_{NL} and the KSS tests will display similar size and/or power performances whenever it is possible to approximate an ESTAR model sufficiently well with an LSTAR model, and viceversa. An ESTAR model can be well approximated by an LSTAR model if most of the data generated by the model lie above the value of the threshold parameter c. Then, only the increasing portion of the transition function matters and this can be well mapped by a monotonically increasing function of LSTAR type. Conversely, an LSTAR model may be well approximated by an ESTAR one provided that the move from one regime to the other is not quick. Thus, for large values of θ, one would expect that the KSS test will perform poorly if the true model is of an LSTAR type.

For sake of simplicity and following Kapetanios et al. (2003), the Monte Carlo simulations set the trend and power parameters to zero. The following DGP serves as the basis for the sample size investigations:

$$y_t = y_{t-1} + u_t, \text{with } u_t = \rho u_{t-1} + \epsilon_t \ , \tag{15}$$

where, as before ϵ_t is standard normally distributed error. The range of possible values for the parameter ρ includes $\{0,\ 0.2,\ 0.5,\ 0.8\}$.

Alternatively, in order to evaluate the power of the tests, the Monte Carlo simulations employ the following LSTAR model:

$$\Delta y_t = \beta y_{t-1}[1 + exp(-\theta y_{t-1})]^{-1} + \epsilon_t \tag{16}$$

where $\epsilon_t \sim N(0, 1)$, $\beta \in$ {-1.9, -1.5, -1.0, -0.1}, and $\theta \in \{0.5,\ 10.0,\ 20.0,\ 100.0\}$. Each experiment uses a 5% nominal size, 20,000 replications, and sample sizes of $T \in$ {50, 100, 200}.

Table (2) shows the size results for the various tests, sample sizes, and degrees of correlation considered.

[Insert Table 2 about here]

The new F_{NL} test exhibits a mild size distortion, especially for the smallest sample size considered. As expected, the size distortion increases with the degree of error correlation. However, for lower correlation levels and higher sample sizes, the size of the F_{NL} test appears closer to the nominal level of 5%. The KSS test tends to under-reject somewhat, while especially for the de-trended data the EG and DF tests appear to over-reject. In contrast, the F_{NL} test performs well in this case.

Next, table (3) presents the power performance of the tests for the three cases.

[Insert Table 3 about here]

In general, for Case 1, the F_{NL} test exhibits lower power than the rest, according to the level of the slope parameter. Thus, in the region of the null (i.e. low levels of β), the KSS and the DF tests perform better. Instead, in the region of the alternative, the EG test has more power than all other tests. However, one can note some differences. For instance, when the slope parameter is close to zero so that the DGP approximates a linear process, the DF test performs equally well and even better than the EG test. The F_{NL} test exhibits better power for higher values of θ, but still less that that of the EG test. Most likely, in these cases the EG test approximates well the DGP and is less parameterized than the F_{NL} test.

Nevertheless, the new test performs better for the demeaned and/or de-trended data. This appears encouraging given that in practice one expects to encounter more often Cases 2 and 3, respectively. Again, when the DGP is approximately linear, there does not appear to be much gain from using a test different than a simple DF one. However, when the DGP behaves more like an asymmetric smooth transition regime, then the F_{NL} test outperforms the other tests considered. Still, in the region of the null the EG and DF tests exhibit slightly higher power, albeit by not much. These conclusions hold true for the de-trended data as well, although the power performance appears lower for all tests.

To further investigate the relative power of the tests under the alternative, I consider the following DGP that corresponds to testing framework (17):

$$\Delta y_t = \delta y_{t-1} + \beta y_{t-1}[1 + exp(-\theta(y_{t-1} - c))]^{-1} + \epsilon_t, \tag{17}$$

where $\delta = 0.1$, and β and θ take the same values as above. Although the theoretical discussion omits this particular case, it poses empirical importance. Under these parameter values, the process is locally explosive (i.e., in the low regime), but still globally geometric ergodic (see Kapetanios et al. (2003)). Table (4) shows the set of results for this particular DGP. The general picture is preserved, although the F_{NL} test displays even higher power in this case.

[Insert Table 4 about here]

4 Empirical Application

The law of one price and the theory of purchasing power parity (denoted PPP hereafter) represent one of the most important topics of international finance and macroeconomics. However, in general empirical research has found limited support for the PPP theory (see Meese and Rogoff (1988), Enders (1988), Taylor (1988)). These early findings are based on the use of augmented DF tests. Explanations of these negative empirical results include the presence of tariff and non-tariff barriers, transportation costs, government regulations, and other restrictions on trade.

However, more recent research suggests that short-run deviations from PPP such as due to volatile exchange rates die away over time, with only half the effect of a temporary departure from the PPP remaining after four years (see Frankel and Rose (1996)). Thus, PPP should hold in the long run and consequently this implies mean reversion of real exchange rates. Moreover, several recent studies find evidence in favor of nonlinear but globally stationary real exchange rates with smooth adjustment (see Sollis et al. (2002), Kapetanios et al. (2003), Bec et al. (2004)). These findings support theoretical models (e.g. Piet et al. (1995)) which suggest that, taking into account the effects of transaction costs, deviations from the law of one price that are nonlinear in nature may arise.

The empirical section of this paper brings further evidence in this direction. Given the nature of the test, the empirical analysis considers the possibility of both symmetric and asymmetric adjustment to the long run PPP relationship. Additionally, it considers both bilateral real exchange rates and real effective exchange rates. The former are computed as the product of the number of foreign currency units per US dollar and the US Producer (Wholesale) Price Index (PPI/WPI) and/or Consumer Price Index (CPI), divided by each nation's WPI/PPI and/or CPI, respectively. The latter (abbreviated as REER) represent a nominal effective exchange rate index adjusted for relative movements in national price or cost indicators of the home, selected countries, and the euro area.

The data comes from the International Financial Statistics (IFS) database and contains quarterly observations beginning in January 1957 until April 2007. This period comprises the fixed exchange rate period (1957-1971) and the flexible exchange rate period (January 1973 - April 2007). The Appendix contains the graphs of the series employed in the analysis.

To get an ampler perspective, the empirical discussion considers both the Kapetanios et al. (2003) test and the augmented DF test along with the new F_{NL} test. Whenever the null of a unit root is rejected, then one can employ the linearity tests of Teräsvirta (1994) to distinguish the type of adjustment (i.e., either symmetric or asymmetric). Table (5) shows the results. All the series under analysis have been demeaned and/or de-trended in line with the theoretical developments of the test. However, note that the PPP theory does not usually support the idea of a trend in real exchange rates. Thus, the discussion here emphasize the results with demeaned data.

Several findings emerge. First, it appears that most real exchange rates display some sort of nonlinear smooth adjustment to a long run mean. Thus, once one controls for this type of nonlinearity, one finds support for the PPP hypothesis. For example, the F_{NL} and/or the KSS tests show that 8 of the 18 bilateral PPI-based real exchange rates considered are nonlinear but globally stationary with smooth adjustment. In contrast, a simple ADF test fails to reject the null of a unit root for these series. The same conclusion characterizes the CPI-based bilateral real exchange rates. Thus, for 9 out of 20 series, the F_{NL} and/or the KSS tests find evidence of global stationarity whereas the ADF test fails to do so. Although more limited, the same evidence emerges for the REER series (i.e. 4 out of 19 series available).

Second, among the series which appear to be nonlinear but globally stationary, some display symmetric while others display asymmetric mean reversion to the long run PPP equilibrium. As mentioned above, the proposed F_{NL} test has power against both LSTAR and ESTAR adjustments. Thus, the advantage of the F_{NL} test lies with the fact that it is able to identify more diverse nonlinear structures. However, given its heavier parametrization, it may lose power when the true source of nonlinearity is of ESTAR type. For instance, the F_{NL} test suggests that Denmark's and Norway's CPI-based real exchange rate are globally stationary while the KSS test fails to do so. Moreover, a simple linearity test confirms this finding. Thus, at least for these two countries the adjustment to the long-run PPP appears asymmetric. However, for Austria, the F_{NL} test fails to reject the null of a unit root while the KSS test does so. Further, a linearity test strengthens the finding of a nonlinear but symmetric adjustment to the long-run mean. In general, whenever the null is rejected, the evidence suggests that most countries display LSTAR-type mean reversion to long-run equilibrium.

Third, with only one exception, the F_{NL} test rejects the null whenever an ADF test does so. Moreover, a linearity test most times rejects the null of a linear process in favor of one with LSTAR-type rather than ESTAR type nonlinearity.

Lastly, there are several instances where although both the F_{NL} test and/or the KSS reject the null of a unit root, a linearity test along Teräsvirta (1994)'s approach, fails to reject the null of linearity. With only exception (i.e. Australia's PPI-based bilateral real exchange rate) this happens when de-trended data are employed. Hence, it appears that in these cases the inclusion of a simple linear time trend is sufficient to capture the nonlinear behavior of the DGP, and therefore the STAR test fails to reject the null of a linear process. As mentioned, one needs to carefully interpret the results when working with de-trended real exchange rate data.

5 Conclusion

This paper makes a three-fold contribution to the literature. Based on the work of Teräsvirta (1994) and Kapetanios et al. (2003), the present paper proposes a new unit root test that has power against nonlinear but globally stationary alternatives, where the adjustment is smooth over time. In contrast to Bec et al. (2004), the new test allows for a higher order Taylor series approximation that has power against more types of nonlinearity.

Second, the paper derives the asymptotic limit distributions of this new test and of a variant of the test, where the threshold parameter is assumed to be zero. In both cases, the asymptotic distribution is free of any nuisance parameters.

Third, the paper tests the validity of the PPP hypothesis using a set of three different real exchange rates: PPI and CPI-based bilateral real exchange rates and real effective exchange rates, respectively. In contrast to a simple DF test, the new test finds that in roughly half of the OECD countries the PPP hypothesis is verified. The empirical discussion shows that the new test should be used together with earlier tests proposed by Teräsvirta (1994) and Kapetanios et al. (2003) to distinguish whether the adjustment to long-run equilibrium is either symmetric or asymmetric over time.

References

Balke, N. S. and Fomby, T. B. (1997). Threshold cointegration. *International Economic Review*, 38(3):627–45.

Bec, F., Salem, M. B., and Carrasco, M. (2004). Detecting mean reversion in real exchange rates from a multiple regime star model. RCER Working Papers 509, University of Rochester - Center for Economic Research (RCER).

Chan, K. S. and Tong, H. (1986). On estimating thresholds in autoregressive models. *Journal of Time Series Analysis*, 7:179–190.

Davies, R. (1987). Hypothesis testing when a nuisance parameter is present under the alternative. *Biometrika*, 74:33–43.

Dickey, D. and Fuller, W. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, 74:427–431.

Enders, W. (1988). ARIMA and cointegration tests of PPP under fixed and flexible exchange rate regimes. *Review of Economics and Statistics*, 70:504–508.

Frankel, J. A. and Rose, A. K. (1996). A panel project on purchasing power parity: Mean reversion within and between countries. *Journal of International Economics*, 40(1-2):209–224.

Kapetanios, G., Shin, Y., and Snell, A. (2003). Testing for a unit root against nonlinear star models. *Journal of Econometrics*, 122(2):359–379.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan, London.

Leybourne, S., Newbold, P., and Vougas, D. (1998). Unit roots and smooth transitions. *Journal of Time Series Analysis*, 19(1):83–97.

Lin, J.-L. and Granger, C. (1994). Forecasting from non-linear models in practice. *Journal of Forecasting*, 13(1):1–9.

Maddala, D. S. (1977). *Econometrics*. McGraw-Hill, New York.

Meese, R. A. and Rogoff, K. (1988). Was it real? the exchange rate-interest differential relation over the modern floating-rate period. *Journal of Finance*, 43(4):933–48.

Perron, P. (1989). The Great Crash, the oil price shock and the unit root hypothesis. *Econometrica*, 57:1361–401.

Phillips, P. and Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75:335–346.

Piet, S., Uppal, R., and Van Hulle, C. (1995). The exchange rate in the presence of transactions costs: Implications for tests of purchasing power parity. *Journal of Finance*, 50:1309–19.

Proietti, T. (2003). Forecasting the us unemployment rate. *Computational Statistics & Data Analysis*, 42(3):451–476.

Sollis, R., Leybourne, S., and Newbold, P. (2002). Tests for symmetric and asymmetric nonlinear mean reversion in real exchange rates. *Journal of Money, Credit and Banking*, 34(3):686–700.

Taylor, M. P. (1988). An empirical examination of long-run purchasing power parity using cointegration techniques. *Applied Economics*, 20(10):1369–81.

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association*, 89:208–18.

Teräsvirta, T. (1998). *Modelling Economic Relationships with Smooth Transition Regressions*, pages 507–552. In: Ullah, A., Giles, D.E (Eds.), Handbook of Applied Economic Studies.Dekker, New York.

Teräsvirta, T. (2006). *Forecasting economic variables with nonlinear models.* Elsevier, Chapter 8 of Handbook of Economic Forecasting, edited by: G. Elliot, C. Granger and A. Timmermann, Amsterdam: New-Holland.

Teräsvirta, T. and Eliasson, A.-C. (2001). Non-linear error correction and the uk demand for broad money, 1878-1993. *Journal of Applied Econometrics*, 16(3):277–288.

Appendix

Proofs

Proof 5.1 *The following results are needed:*

$$\frac{1}{T^3}\sum_{t=1}^{T} y_{t-1}^4 = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{\sqrt{T}}y_{t-1}\right)^4 = \frac{1}{T}\sum_{t=1}^{T}\int_{\frac{j-1}{T}}^{\frac{j}{T}}\left(\frac{1}{\sqrt{T}}Y_T(r)dr\right)^4 \to_d \int_0^1 B(r)^4 dr = \sigma^4\int_0^1 W(r)^4 dr$$

where $\frac{j-1}{T} \leq t < \frac{j}{T}$, $Y_T(r) = \frac{S_{j-1}}{\sqrt{T}} = \frac{\sum_{t=1}^{j-1}\epsilon_t}{\sqrt{T}}$, *and* $B(r) = BM(\sigma^2) = \sigma W(r)$. *Similarly, we have:*

$$\frac{1}{T^{\frac{7}{2}}}\sum_{t=1}^{T} y_{t-1}^5 \to_d \int_0^1 B(r)^5 dr = \sigma^5\int_0^1 W(r)^5 dr$$

$$\frac{1}{T^4}\sum_{t=1}^{T} y_{t-1}^6 \to_d \int_0^1 B(r)^6 dr = \sigma^6\int_0^1 W(r)^6 dr$$

$$\frac{1}{T^{\frac{9}{2}}}\sum_{t=1}^{T} y_{t-1}^7 \to_d \int_0^1 B(r)^7 dr = \sigma^7\int_0^1 W(r)^7 dr$$

$$\frac{1}{T^5}\sum_{t=1}^{T} y_{t-1}^8 \to_d \int_0^1 B(r)^8 dr = \sigma^8\int_0^1 W(r)^8 dr$$

Now using directly the continuous mapping theorem, Itô's formula, and the weak convergence of stochastic integrals one obtains that:

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} y_{t-1}^2\epsilon_t \to_d \int_0^1 B(r)^2 dB(r) = \sigma^3\left(\frac{1}{3}\int_0^1 W(1)^3 - \int_0^1 W(r)dr\right)$$

$$\frac{1}{T^2}\sum_{t=1}^{T} y_{t-1}^3\epsilon_t \to_d \int_0^1 B(r)^3 dB(r) = \sigma^4\left(\frac{1}{4}\int_0^1 W(1)^4 - \frac{3}{2}\int_0^1 W(r)^2 dr\right)$$

$$\frac{1}{T^{\frac{5}{2}}}\sum_{t=1}^{T} y_{t-1}^4\epsilon_t \to_d \int_0^1 B(r)^4 dB(r) = \sigma^5\left(\frac{1}{5}\int_0^1 W(1)^5 - 2\int_0^1 W(r)^3 dr\right)$$

It is clear that the estimators have different convergence rates. Thus, the least squares estimators need to be scaled using the following scaling matrix: $\mathbf{D_T} = diag(T^{3/2}, T^2, T^{5/2})$.

Denote by $\hat{\gamma}_T = (\hat{\gamma}_1, \gamma_2, \hat{\gamma}_3)$ *and by* $\mathbf{X_T} = (y_{t-1}^2, y_{t-1}^3, y_{t-1}^4)'$. *Then, we have that:*

$$\mathbf{D_T}(\hat{\gamma}_T - \gamma) = \left[\mathbf{D_T}^{-1}\left(\sum_{t=1}^{T} X_t X_t'\right)\mathbf{D_T}^{-1}\right]^{-1}\left[\mathbf{D_T}^{-1}\left(\sum_{t=1}^{T} X_t\epsilon_t\right)\right]$$

After some algebra one gets that

$$\mathbf{D_T}(\hat{\gamma}_T - \gamma) \to_L \frac{1}{\sigma}(\Gamma\mathbf{Q}\Gamma)^{-1}(\Gamma\mathbf{v})$$

where

$$\mathbf{v} = \begin{bmatrix} \frac{1}{3}W(1)^3 - \int_0^1 W(r)dr \\ \frac{1}{4}W(1)^4 - \frac{3}{2}\int_0^1 W(r)^2dr \\ \frac{1}{5}W(1)^5 - 2\int_0^1 W(r)^3dr \end{bmatrix}$$

and

$$\mathbf{Q} = \begin{bmatrix} \int_0^1 W(r)^4dr & \int_0^1 W(r)^5dr & \int_0^1 W(r)^6dr \\ \int_0^1 W(r)^5dr & \int_0^1 W(r)^6dr & \int_0^1 W(r)^7dr \\ \int_0^1 W(r)^6dr & \int_0^1 W(r)^7dr & \int_0^1 W(r)^8dr \end{bmatrix}$$

and $\Gamma = diag(1, \sigma, \sigma^2)$.

A general F-test tests the restriction: $\mathbf{R}\gamma = \mathbf{r}$, *where* $\mathbf{R} = \mathbf{I_3}$ *and* $\mathbf{r} = [0,0,0]'$. *Our test has then the following representation:*

$$F_{NL} = (\hat{\gamma}_T - \gamma)'(\mathbf{RD_T})' \left[\hat{\sigma}_T^2\mathbf{D_T}\mathbf{R}\left(\sum X_T X_T'\right)^{-1}\mathbf{D_T}\mathbf{R}'\right]^{-1}\mathbf{RD_T}(\hat{\gamma}_T - \gamma)$$

which has the limiting distribution:

$$F_{NL} \to_L \left((\Gamma\mathbf{Q}\Gamma)^{-1}(\Gamma\mathbf{v})\right)'\left((\Gamma\mathbf{Q}\Gamma)^{-1}\right)^{-1}\left((\Gamma\mathbf{Q}\Gamma)^{-1}(\Gamma\mathbf{v})\right) = \mathbf{v}'\mathbf{Q}^{-1}\mathbf{v}$$

It is easy to show that the least squares estimate $\hat{\sigma}_T^2$ *converges in probability to* σ_T^2 *under the null. Next, under the alternative,* Δy_t, y_{t-1}^2, y_{t-1}^3, *and* y_{t-1}^4 *are I(0) and it is straightforward to show that the terms involved are* $O_p(1)$. *Hence, the* F_{NL} *statistic diverges to infinity at the rate* $O_p(\mathbf{D_T})$.

Proof 5.2 *The resulting limit follows similarly as in proof (5.1), with the following replacements needed:* $\bar{\mathbf{D}}_\mathbf{T} = diag(T^{3/2}, T^{5/2})$ *and* $\Gamma = diag(1, \sigma^2)$.

Proof 5.3 *Define the* $T \times p$ *matrix* $\mathbf{Z} = (\Delta y_{-1}, \ldots, \Delta y_{-p})$, *and the* $T \times T$ *idempotent matrix* $\mathbf{M}_T = \mathbf{I}_T$ - $\mathbf{Z}$ $(\mathbf{Z}'\ \mathbf{Z})^{-1}$ $\mathbf{Z}'$. *Now,*

$$\hat{\sigma}_T^2 = \frac{1}{T}\epsilon'\mathbf{M}_T\epsilon = \frac{1}{T}\epsilon'\epsilon + o_p(1) \to_p \sigma^2,$$

where $\epsilon = (\epsilon_1, \ldots, \epsilon_T)$. *Next, similar to Kapetanios et al. (2003) it is straightforward* *to show under the null that*

$$\frac{1}{T^3} y_{-1}^{2'} \mathbf{M}_T y_{-1}^2 = \frac{1}{T^3} y_{-1}^{2'} y_{-1}^2 + o_p(1) \rightarrow_d \int_0^1 B(r)^4 dr = \lambda^4 \int_0^1 W(r)^4 dr$$

$$\frac{1}{T^{7/2}} y_{-1}^{5/2'} \mathbf{M}_T y_{-1}^{5/2} = \frac{1}{T^7/2} y_{-1}^{5/2'} y_{-1}^{5/2} + o_p(1) \rightarrow_d \int_0^1 B(r)^5 dr = \lambda^5 \int_0^1 W(r)^5 dr$$

$$\frac{1}{T^4} y_{-1}^{3'} \mathbf{M}_T y_{-1}^3 = \frac{1}{T^4} y_{-1}^{3'} y_{-1}^3 + o_p(1) \rightarrow_d \int_0^1 B(r)^6 dr = \lambda^6 \int_0^1 W(r)^6 dr$$

$$\frac{1}{T^{9/2}} y_{-1}^{7/2'} \mathbf{M}_T y_{-1}^{7/2} = \frac{1}{T^9/2} y_{-1}^{7/2'} y_{-1}^{7/2} + o_p(1) \rightarrow_d \int_0^1 B(r)^7 dr = \lambda^7 \int_0^1 W(r)^7 dr$$

$$\frac{1}{T^5} y_{-1}^{4'} \mathbf{M}_T y_{-1}^4 = \frac{1}{T^5} y_{-1}^{4'} y_{-1}^4 + o_p(1) \rightarrow_d \int_0^1 B(r)^8 dr = \lambda^8 \int_0^1 W(r)^8 dr$$

Analogously, we have:

$$\frac{1}{T^{3/2}} y_{-1}^{2'} \mathbf{M}_T \epsilon = \frac{1}{T^{3/2}} y_{-1}^{2'} \epsilon + o_p(1) \rightarrow_d \int_0^1 B(r)^2 dB(r) = \lambda^{3/2} \sigma^{3/2} \left(\frac{1}{3} \int_0^1 W(1)^3 - \int_0^1 W(r) dr \right)$$

$$\frac{1}{T^2} y_{-1}^{3'} \mathbf{M}_T \epsilon = \frac{1}{T^2} y_{-1}^{3'} \mathbf{M}_T \epsilon + o_p(1) \rightarrow_d \int_0^1 B(r)^3 dB(r) = \lambda^2 \sigma^2 \left(\frac{1}{4} \int_0^1 W(1)^4 - \frac{3}{2} \int_0^1 W(r)^2 dr \right)$$

$$\frac{1}{T^{5/2}} y_{-1}^{4'} \mathbf{M}_T \epsilon = \frac{1}{T^{5/2}} y_{-1}^{4'} \epsilon + o_p(1) \rightarrow_d \int_0^1 B(r)^4 dB(r) = \lambda^{5/2} \sigma^{5/2} \left(\frac{1}{5} \int_0^1 W(1)^5 - 2 \int_0^1 W(r)^3 dr \right)$$

where $y_{-1}^{\kappa} = (y_0^{\kappa}, y_1^{\kappa}, \ldots, y_{T-1}^{\kappa})'$ *for* $\kappa \in \{2, 3, 4\}$ *and* λ *is the long-run variance of* Δy_t *under the null. Using this set of results, it is straightforward to show that the limiting distribution obtained under the assumption of iid errors is preserved. Lastly, following the same reasoning as in Proof 5.1 one can conclude that the* F_{NL} *test is consistent under the alternative as well.*

Proof 5.4 *The resulting limit follows similarly as in proof (5.3), with the following replacements needed:* $\mathbf{\bar{D}_T} = diag(T^{3/2}, T^{5/2})$ *and* $\Gamma = diag(1, \sigma^2)$*.*

Tables

Table 1: Asymptotic critical values of the F_{NL} and $\bar{F}_{NL}$ tests

Fractile (%)	Case 1	Case 2	Case 3
F_{NL}			
10	3.05	3.30	4.05
5	3.64	3.87	4.72
1	4.92	5.16	6.08
$\bar{F}_{NL}$	Case 1	Case 2	Case 3
10	3.67	2.66	1.90
5	4.51	3.42	2.46
1	6.40	5.06	3.73

Note: Case 1, Case 2, and Case 3 refer to the underlying model with raw data, demeaned, and de-trended data, respectively.

Table 2: The size of alternative tests

	Case 1				Case 2				Case 3			
	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF
ρ=**0**												
T= 50	0.054	0.043	0.046	0.052	0.053	0.046	0.060	0.060	0.050	0.048	0.066	0.070
T=100	0.051	0.046	0.042	0.048	0.049	0.047	0.051	0.053	0.046	0.046	0.058	0.058
T=200	0.053	0.049	0.046	0.050	0.053	0.049	0.051	0.052	0.047	0.048	0.055	0.056
ρ=**0.2**												
T= 50	0.064	0.046	0.052	0.051	0.061	0.049	0.056	0.054	0.056	0.056	0.061	0.066
T=100	0.059	0.048	0.047	0.050	0.056	0.049	0.054	0.057	0.052	0.053	0.058	0.058
T=200	0.058	0.051	0.045	0.051	0.055	0.049	0.050	0.050	0.051	0.050	0.052	0.053
ρ=**0.5**												
T= 50	0.071	0.049	0.054	0.049	0.071	0.054	0.059	0.056	0.065	0.064	0.064	0.067
T=100	0.067	0.052	0.052	0.048	0.060	0.048	0.050	0.052	0.060	0.058	0.057	0.059
T=200	0.059	0.049	0.051	0.048	0.055	0.051	0.051	0.052	0.053	0.053	0.052	0.057
ρ=**0.8**												
T= 50	0.085	0.051	0.071	0.051	0.074	0.055	0.059	0.057	0.079	0.075	0.064	0.069
T=100	0.072	0.052	0.061	0.051	0.065	0.050	0.056	0.054	0.063	0.060	0.056	0.057
T=200	0.065	0.049	0.057	0.049	0.060	0.049	0.052	0.052	0.057	0.052	0.055	0.055

Table 3: The power of alternative tests against the hypothesis of global LSTAR stationarity

	$\theta = 0.5$				$\theta = 10.0$				$\theta = 20.0$				$\theta = 100.0$			
	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF
(a) Case 1																
β**=-1.9**																
T= 50	0.974	0.899	0.999	0.999	0.216	0.034	0.879	0.110	0.208	0.034	0.878	0.107	0.247	0.033	0.688	0.109
T=100	0.999	0.970	1.0	0.999	0.130	0.033	0.768	0.096	0.126	0.032	0.761	0.094	0.173	0.033	0.669	0.083
T=200	0.999	0.989	1.0	0.999	0.072	0.034	0.816	0.078	0.069	0.034	0.813	0.077	0.158	0.017	0.956	0.043
β**=-1.5**																
T= 50	0.868	0.803	0.997	0.993	0.205	0.047	0.582	0.147	0.218	0.046	0.575	0.144	0.214	0.047	0.807	0.106
T=100	0.991	0.919	0.999	0.998	0.139	0.037	0.718	0.110	0.139	0.037	0.681	0.109	0.181	0.035	0.755	0.108
T=200	0.997	0.962	1.0	0.999	0.073	0.037	0.789	0.085	0.070	0.038	0.794	0.084	0.152	0.017	0.629	0.045
β**=-1.0**																
T= 50	0.597	0.638	0.925	0.918	0.206	0.070	0.652	0.139	0.188	0.069	0.444	0.137	0.176	0.068	0.474	0.155
T=100	0.900	0.755	0.998	0.977	0.162	0.049	0.618	0.113	0.160	0.049	0.607	0.111	0.187	0.047	0.543	0.112
T=200	0.959	0.825	0.999	0.986	0.098	0.037	0.704	0.107	0.100	0.044	0.702	0.106	0.129	0.022	0.531	0.057
β**=-0.1**																
T= 50	0.058	0.115	0.071	0.140	0.062	0.125	0.076	0.123	0.056	0.125	0.074	0.123	0.062	0.124	0.086	0.118
T=100	0.082	0.192	0.130	0.312	0.069	0.153	0.113	0.212	0.079	0.153	0.114	0.211	0.086	0.161	0.124	0.189
T=200	0.162	0.254	0.271	0.375	0.137	0.133	0.189	0.197	0.127	0.134	0.186	0.197	0.102	0.108	0.161	0.126
(b) Case 2																
β**=-1.9**																
T= 50	0.975	0.911	0.999	0.997	0.401	0.360	0.266	0.233	0.464	0.301	0.206	0.246	0.377	0.268	0.233	0.184
T=100	1.0	0.974	0.999	0.999	0.490	0.382	0.289	0.223	0.459	0.316	0.231	0.240	0.515	0.208	0.280	0.174
T=200	1.0	0.988	1.0	1.0	0.510	0.357	0.284	0.186	0.566	0.352	0.291	0.220	0.463	0.171	0.332	0.133
β**=-1.5**																
T= 50	0.894	0.817	0.982	0.977	0.309	0.277	0.215	0.188	0.387	0.271	0.202	0.157	0.419	0.251	0.212	0.160
T=100	0.999	0.927	0.999	0.996	0.484	0.313	0.251	0.233	0.464	0.256	0.186	0.197	0.410	0.203	0.290	0.171
T=200	0.999	0.963	0.999	0.999	0.439	0.269	0.212	0.182	0.494	0.311	0.258	0.218	0.460	0.118	0.268	0.106
β**=-1.0**																
T= 50	0.604	0.590	0.822	0.824	0.291	0.241	0.216	0.158	0.282	0.231	0.198	0.196	0.306	0.171	0.219	0.179
T=100	0.955	0.776	0.978	0.958	0.389	0.235	0.197	0.162	0.312	0.255	0.224	0.179	0.386	0.176	0.253	0.163
T=200	0.996	0.838	0.989	0.980	0.439	0.259	0.230	0.207	0.407	0.255	0.227	0.207	0.395	0.086	0.216	0.102
β**=-0.1**																
T= 50	0.062	0.066	0.072	0.086	0.055	0.065	0.076	0.080	0.056	0.066	0.075	0.077	0.068	0.056	0.077	0.067
T=100	0.075	0.094	0.094	0.131	0.075	0.081	0.088	0.101	0.073	0.086	0.094	0.089	0.083	0.060	0.084	0.073
T=200	0.130	0.150	0.171	0.199	0.110	0.105	0.106	0.115	0.103	0.112	0.116	0.112	0.091	0.059	0.095	0.089
(c) Case 3																
β**=-1.9**																
T= 50	0.946	0.870	0.996	0.993	0.345	0.317	0.235	0.219	0.300	0.277	0.195	0.230	0.315	0.281	0.213	0.210
T=100	0.999	0.965	1.0	0.999	0.393	0.326	0.241	0.204	0.351	0.283	0.201	0.211	0.365	0.273	0.240	0.231
T=200	1.0	0.985	1.0	1.0	0.398	0.294	0.231	0.170	0.399	0.295	0.234	0.193	0.382	0.262	0.265	0.205
β**=-1.5**																
T= 50	0.798	0.741	0.959	0.954	0.261	0.243	0.194	0.181	0.259	0.241	0.189	0.160	0.285	0.244	0.200	0.187
T=100	0.992	0.903	0.997	0.996	0.329	0.266	0.214	0.204	0.285	0.238	0.174	0.179	0.314	0.252	0.241	0.217
T=200	0.999	0.953	0.999	0.999	0.321	0.232	0.187	0.168	0.342	0.249	0.206	0.184	0.315	0.198	0.222	0.171
β**=-1.0**																
T= 50	0.468	0.457	0.726	0.720	0.193	0.185	0.188	0.156	0.187	0.179	0.177	0.179	0.184	0.164	0.187	0.189
T=100	0.868	0.717	0.955	0.935	0.228	0.193	0.172	0.149	0.245	0.300	0.189	0.162	0.221	0.192	0.206	0.190
T=200	0.982	0.810	0.984	0.975	0.282	0.204	0.188	0.174	0.274	0.198	0.182	0.172	0.236	0.146	0.185	0.163
β**=-0.1**																
T= 50	0.055	0.057	0.080	0.088	0.057	0.061	0.083	0.084	0.056	0.060	0.082	0.081	0.059	0.061	0.085	0.092
T=100	0.066	0.073	0.090	0.104	0.066	0.064	0.081	0.087	0.062	0.066	0.085	0.085	0.067	0.069	0.090	0.094
T=200	0.099	0.104	0.136	0.145	0.077	0.080	0.094	0.097	0.078	0.083	0.099	0.097	0.076	0.073	0.099	0.107

Table 4: The power of alternative tests against the hypothesis of global LSTAR stationarity: DGP2

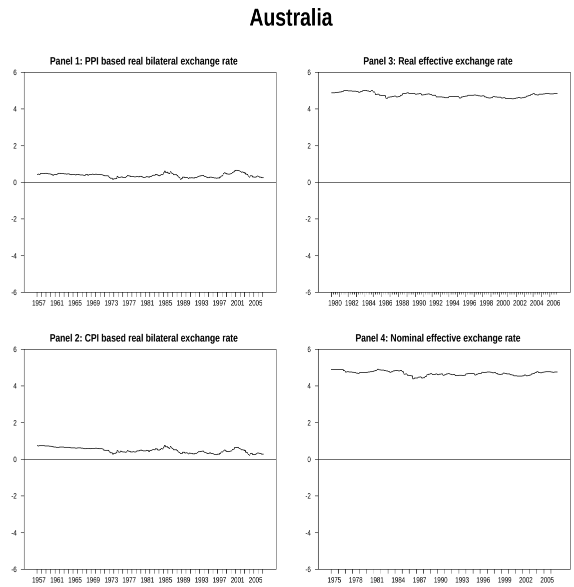
	$\theta = 0.5$				$\theta = 10.0$				$\theta = 20.0$				$\theta = 100.0$			
	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF	F_{NL}	KSS	EG	ADF
(a) Case 1																
β=**-1.9**																
T= 50	0.921	0.783	0.999	0.979	0.948	0.003	0.988	0.009	0.947	0.023	0.988	0.009	0.168	0.003	0.663	0.009
T=100	0.991	0.873	1.0	0.975	0.999	0.001	1.0	0.001	1.0	0.001	1.0	0.001	0.162	0.0	0.523	0.001
T=200	0.998	0.893	1.0	0.947	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	0.154	0.0	0.526	0.001
β=**-1.5**																
T= 50	0.753	0.617	0.975	0.900	0.939	0.005	0.988	0.015	0.944	0.005	0.985	0.010	0.166	0.004	0.496	0.009
T=100	0.961	0.705	1.0	0.897	0.999	0.001	1.0	0.001	0.999	0.001	1.0	0.001	0.173	0.001	0.825	0.001
T=200	0.992	0.690	1.0	0.796	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	0.158	0.0	0.465	0.001
β=**-1.0**																
T= 50	0.533	0.412	0.831	0.638	0.926	0.017	0.944	0.017	0.939	0.010	0.976	0.023	0.172	0.080	0.402	0.026
T=100	0.880	0.372	0.998	0.553	0.999	0.001	1.0	0.001	0.999	0.001	1.0	0.001	0.165	0.001	0.394	0.001
T=200	0.986	0.244	1.0	0.316	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	0.172	0.0	0.419	0.001
β=**-0.2**																
T= 50	0.722	0.049	0.574	0.051	0.711	0.051	0.713	0.037	0.807	0.051	0.652	0.031	0.122	0.028	0.111	0.030
T=100	0.875	0.045	0.850	0.067	0.953	0.017	0.949	0.021	0.946	0.021	0.962	0.016	0.178	0.015	0.197	0.017
T=200	0.989	0.016	0.990	0.019	0.999	0.001	0.999	0.001	0.999	0.001	0.999	0.001	0.139	0.001	0.152	0.003
(b) Case 2																
β=**-1.9**																
T= 50	0.940	0.831	0.987	0.967	0.972	0.016	0.960	0.013	0.974	0.012	0.960	0.013	0.229	0.025	0.150	0.014
T=100	1.0	0.902	1.0	0.973	1.0	0.001	1.0	0.001	1.0	0.001	0.999	0.001	0.299	0.014	0.156	0.006
T=200	1.0	0.904	1.0	0.946	1.0	0.0	1.0	0.0	1.0	0.0	0.999	0.0	0.254	0.030	0.211	0.005
β=**-1.5**																
T= 50	0.780	0.662	0.900	0.842	0.964	0.017	0.948	0.015	0.964	0.016	0.949	0.010	0.249	0.025	0.138	0.011
T=100	0.996	0.756	0.997	0.889	1.0	0.001	1.0	0.001	1.0	0.001	0.998	0.001	0.218	0.014	0.180	0.03
T=200	1.0	0.715	1.0	0.795	1.0	0.0	1.0	0.0	1.0	0.0	0.999	0.0	0.272	0.009	0.160	0.001
β=**-1.0**																
T= 50	0.547	0.371	0.683	0.512	0.935	0.021	0.924	0.014	0.936	0.018	0.925	0.019	0.257	0.016	0.147	0.021
T=100	0.942	0.407	0.871	0.530	1.0	0.001	0.999	0.001	1.0	0.001	0.999	0.001	0.181	0.008	0.173	0.003
T=200	1.0	0.259	1.0	0.313	1.0	0.0	1.0	0.0	1.0	0.0	1.0	0.0	0.204	0.004	0.133	0.002
β=**-0.2**																
T= 50	0.499	0.030	0.625	0.047	0.812	0.025	0.656	0.023	0.775	0.026	0.676	0.020	0.129	0.017	0.102	0.020
T=100	0.888	0.024	0.889	0.049	0.950	0.008	0.958	0.013	0.952	0.010	0.854	0.011	0.136	0.009	0.098	0.013
T=200	0.988	0.010	0.992	0.015	0.998	0.001	0.999	0.001	0.998	0.001	0.999	0.001	0.134	0.003	0.084	0.002
(c) Case 3																
β=**-1.9**																
T= 50	0.870	0.777	0.960	0.952	0.245	0.017	0.114	0.013	0.057	0.013	0.019	0.012	0.151	0.110	0.101	0.105
T=100	0.994	0.888	0.995	0.971	0.411	0.001	0.303	0.001	0.091	0.001	0.053	0.001	0.134	0.098	0.092	0.106
T=200	1.0	0.896	1.0	0.947	0.412	0.0	0.305	0.0	0.108	0.001	0.064	0.001	0.156	0.121	0.112	0.105
β=**-1.5**																
T= 50	0.645	0.578	0.815	0.786	0.245	0.017	0.102	0.014	0.048	0.016	0.027	0.009	0.117	0.091	0.088	0.091
T=100	0.961	0.726	0.978	0.883	0.422	0.001	0.312	0.001	0.067	0.001	0.037	0.001	0.118	0.089	0.092	0.093
T=200	0.999	0.698	0.998	0.795	0.322	0.0	0.233	0.0	0.121	0.001	0.073	0.001	0.122	0.081	0.081	0.081
β=**-1.0**																
T= 50	0.370	0.275	0.482	0.421	0.189	0.018	0.114	0.014	0.059	0.016	0.030	0.018	0.095	0.067	0.079	0.089
T=100	0.822	0.364	0.892	0.500	0.405	0.001	0.296	0.001	0.144	0.001	0.089	0.001	0.086	0.064	0.076	0.075
T=200	0.993	0.248	0.994	0.313	0.456	0.0	0.339	0.0	0.138	0.001	0.084	0.001	0.086	0.059	0.068	0.072
β=**-0.2**																
T= 50	0.450	0.024	0.342	0.042	0.151	0.023	0.088	0.022	0.031	0.022	0.039	0.018	0.063	0.055	0.068	0.081
T=100	0.853	0.015	0.855	0.035	0.543	0.005	0.420	0.010	0.247	0.007	0.155	0.006	0.069	0.049	0.062	0.067
T=200	0.984	0.007	0.987	0.012	0.465	0.001	0.360	0.001	0.281	0.001	0.192	0.001	0.053	0.050	0.062	0.047

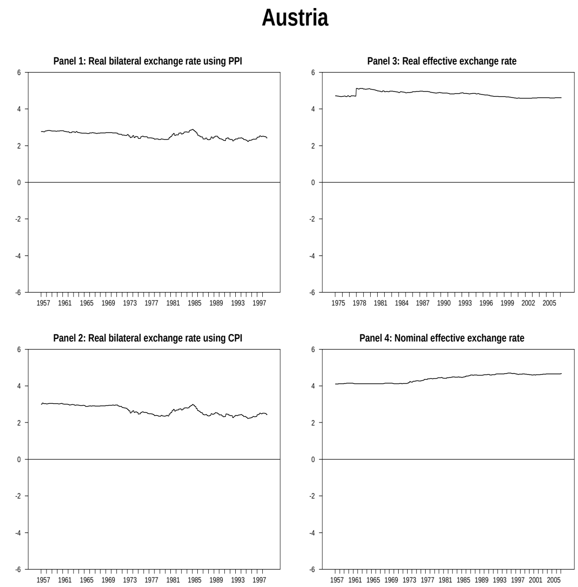
Table 5: Nonlinearity Tests for Bilateral and Real Effective Exchange Rates

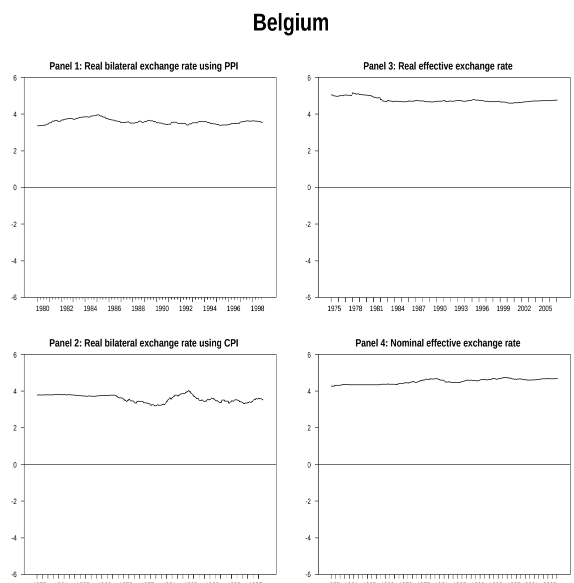
	RER(PPI/WPI)				RER(CPI)				REER			
Country	F_{NL} Test	STAR Test	KSS Test	ADF Test	F_{NL} Test	STAR Test	KSS Test	ADF Test	F_{NL} Test	STAR Test	KSS Test	ADF Test
Australia												
1957:1-2007:4	4.14**	1.95	-2.75*	-2.61	3.30*	2.78*	-2.79*	-2.07	1.46	0.57	-1.81	-1.84
Austria												
1957:1-2007:4	2.80	2.67**†	-2.66*	-1.93	2.18	1.92	-2.20	-1.62	0.42	0.56	-1.07	-1.29
Belgium												
1957:1-2007:4	2.35	1.08	-1.57	-2.21	2.10	1.70	-2.32	-1.99	2.20	0.94	-1.99	-1.96
Canada												
1957:1-2007:4	2.20	0.37	-2.33	-1.41	1.90	0.53	-2.38	-1.60	4.81**	7.06***	-0.85	-1.82
Denmark												
1957:1-2007:4	4.27*‡	4.18***	-2.83*	-1.92	4.04**	3.90***	-1.94	-1.90	0.39	0.81	-0.73	-1.23
Finland												
1957:1-2007:4	4.09*‡	0.49	-3.13*‡	-2.41	4.19*‡	0.21	-3.11*‡	-2.24	9.21***	9.88***	-4.78***	-4.08***‡
France												
1957:1-2007:4	-	-	-	-	3.16	2.29*†	-2.87*	-2.46	8.10***‡	0.36	-4.37***‡	-2.74‡
Germany												
1957:1-2007:4	-	-	-	-	2.63	3.71**	-2.67*	-2.05	0.97	0.10	-1.67	-1.35
Greece												
1957:1-2007:4	4.11**‡	3.23**	-1.46‡	-2.11‡	1.84	0.29	-1.10	-1.79	7.16***‡	0.81	-3.37*‡	-2.22‡
Ireland												
1957:1-2007:4	1.45	2.40**	-1.42	-1.09	1.50	2.57*	-1.35	-1.04	-	-	-	
Italy												
1957:1-2007:4	2.66	4.34**†	-1.91	-1.70	7.37***‡	5.12***	-3.87**‡	-3.01‡	3.99**	4.43**	-1.97	-1.41
Japan												
1957:1-2007:4	2.54	0.68	-2.50	-2.28	2.38	1.52	-1.70	-1.71	2.24	1.22	-2.35	-1.68
Luxembourg												
1957:1-2007:4	2.84	0.90	-1.76	-2.19	2.52	1.06	-2.05	-2.17	2.22	1.17	-2.36	-1.53
Netherlands												
1957:1-2007:4	2.59	1.83	-2.58	-1.93	1.55	1.68	-1.97	-1.81	2.30	1.32	-2.33	-1.80
Norway												
1957:1-2007:4	3.47*	3.32**†	-2.86*	-2.12	6.22***	6.36***	-2.19	-1.94	0.60	0.28	-2.34	-0.16
New Zealand												
1957:1-2007:4	7.11***	4.86***	-4.53**	-2.59*	2.21	3.73**	-1.67	-1.21	2.41	0.02	-1.92	-2.28
Spain												
1957:1-2007:4	2.35	0.35	-2.42	-2.69*	2.75	1.42	-2.24	-1.60	2.76	1.38	-2.28	-2.05
Sweden												
1957:1-2007:4	4.75**	3.15**	-2.90*	-2.56*	5.56***	7.19***†	-2.99**	-2.23	6.78***‡	0.16	-4.19**‡	-3.13*‡
Switzerland												
1957:1-2007:4	4.08*‡	1.64	-3.40**‡	-2.26	5.31**‡	1.24	-1.82	-1.65	5.27**‡	0.10	-3.43**‡	-3.17*‡
UK												
1957:1-2007:4	4.51**	5.38**†	-3.12**	-1.94	4.42**	6.37***†	-3.50**	-3.30*‡	3.20	2.48*	-1.57	-1.40

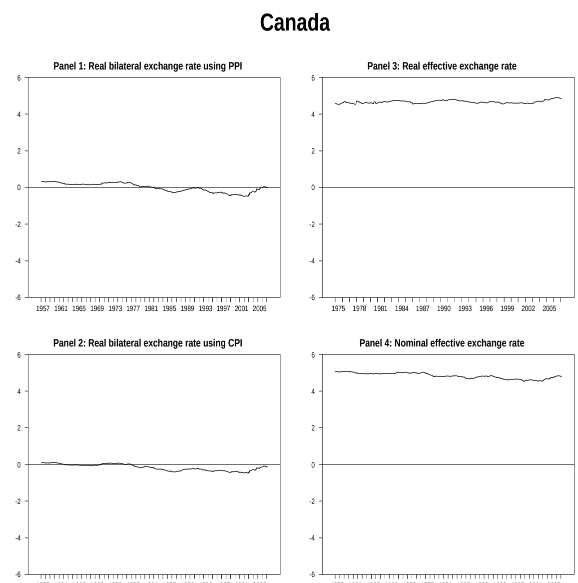
* Significance at the 10% level; ** Significance at the 5% level; *** Significance at the 1% level; The KSS critical values for the demeaned data only are -2.66 and -2.93 for the 10% and 5% significance levels, respectively; The KSS critical values for the demeaned and de-trended data are -3.13 and -3.40 for the 10% and 5% significance levels, respectively; The † symbol suggests that the ESTAR form is preferred over the LSTAR one when the STAR statistic is significant; The ‡ symbol denotes the nonlinear but globally trend stationarity property of the series; Otherwise when the KSS test statistic is significant, it denotes the nonlinear but globally level stationarity property of the series; ADF critical values: 1%= -3.446, 5%= -2.868, 10%= -2.570.

Graphs of Exchange Rates

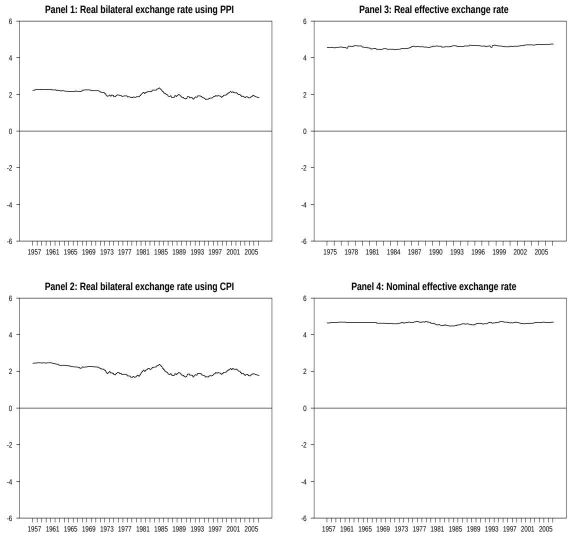




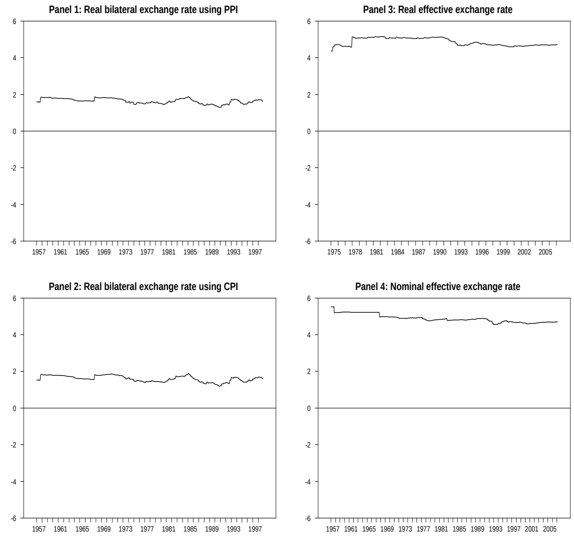




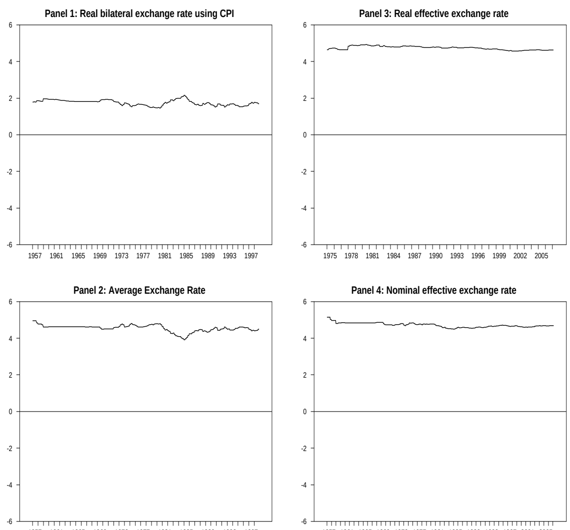
Denmark



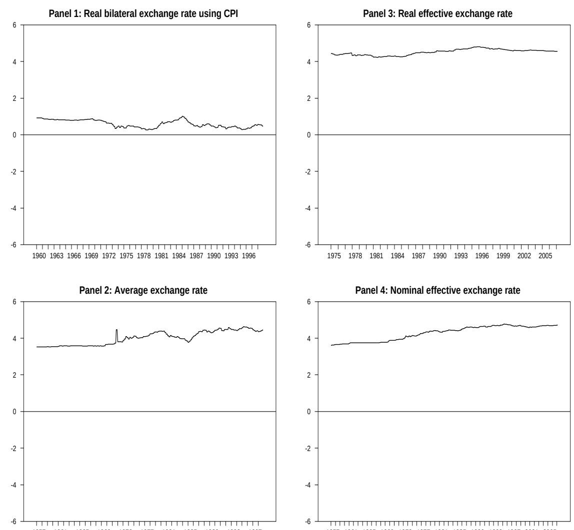
Finland



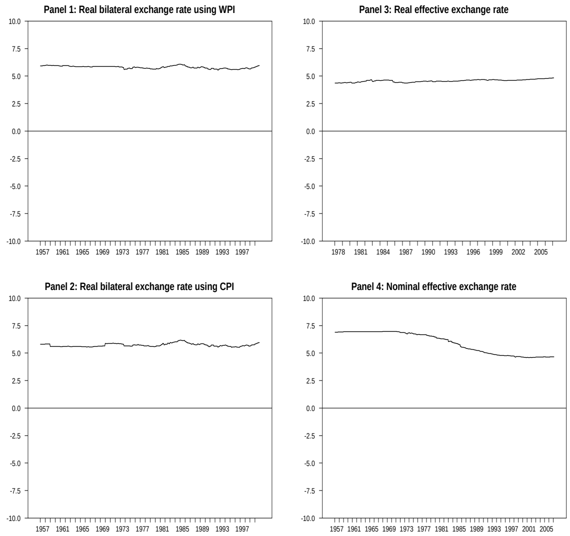
France



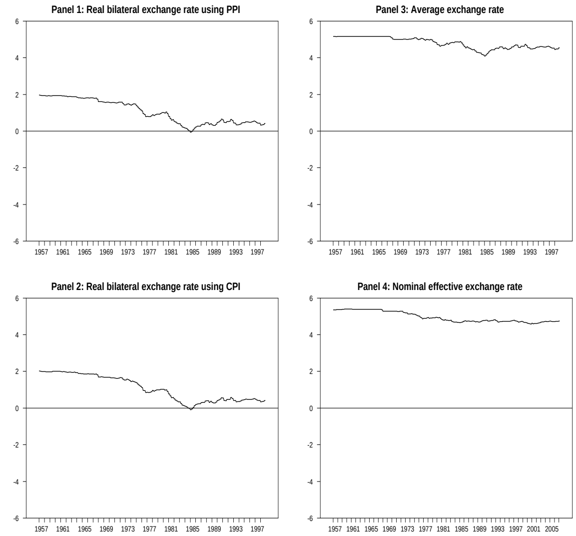
Germany



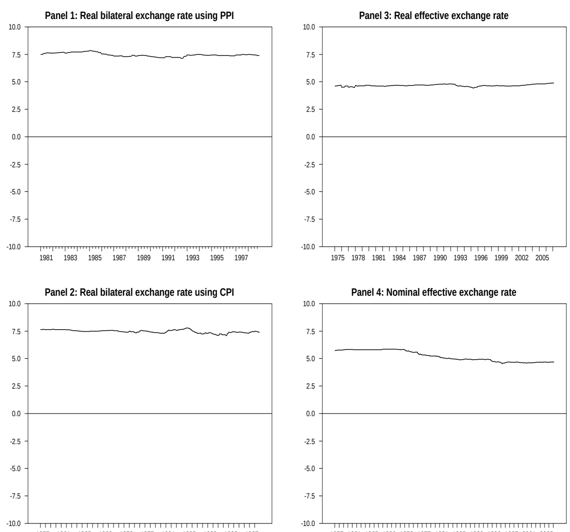
Greece



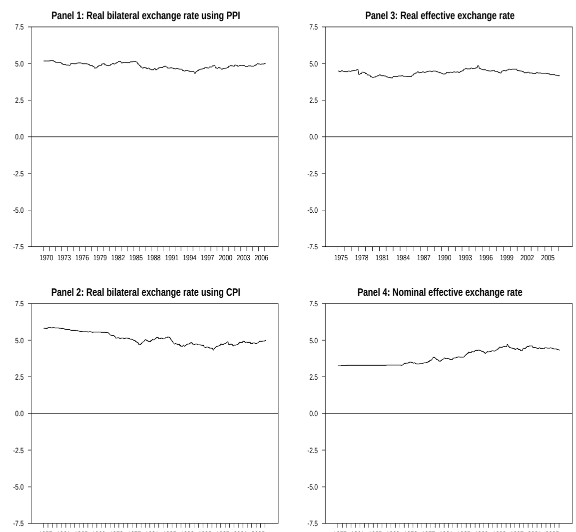
Ireland



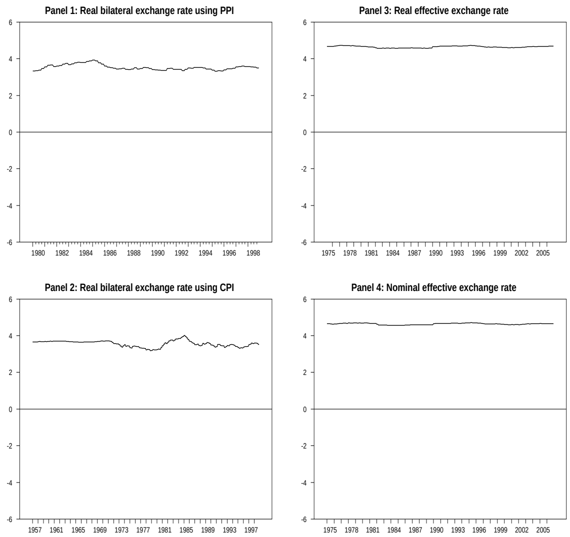
Italy



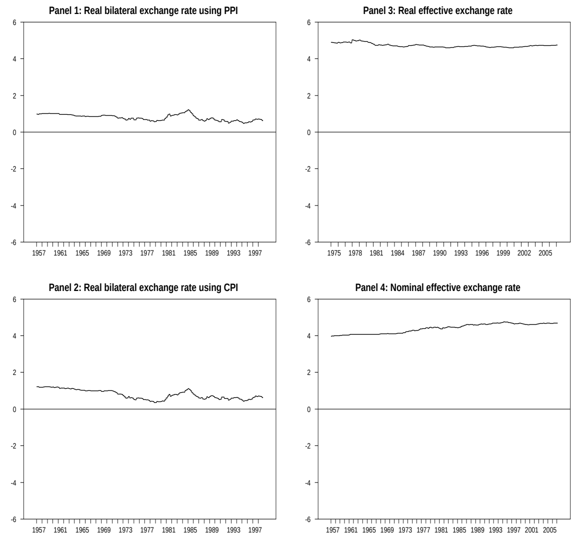
Japan



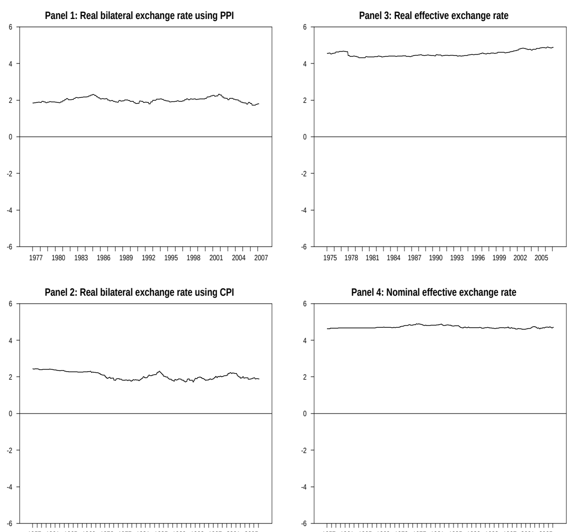
Luxembourg



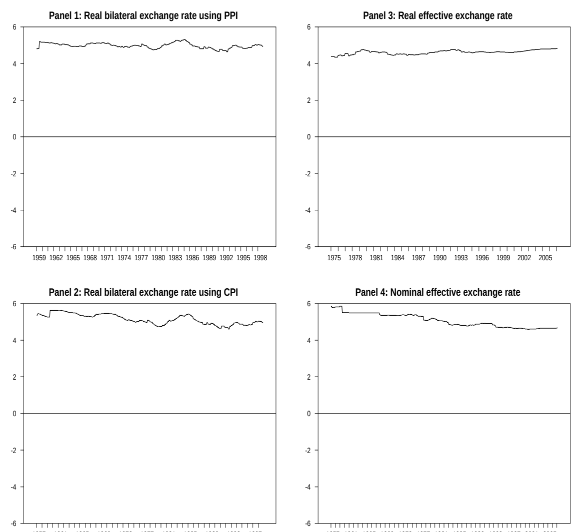
Netehrlands



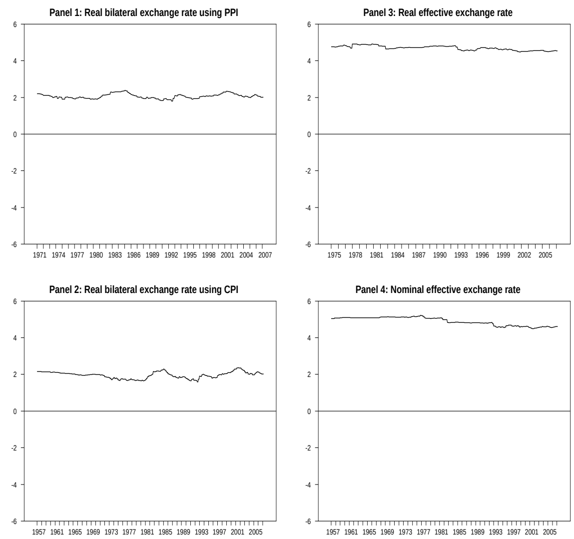
Norway



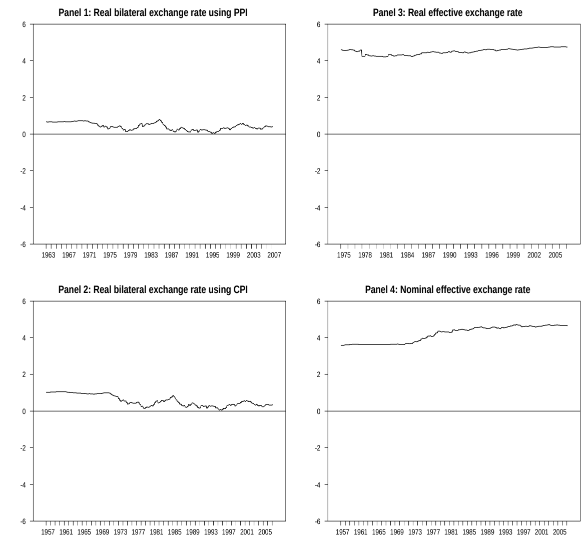
Spain



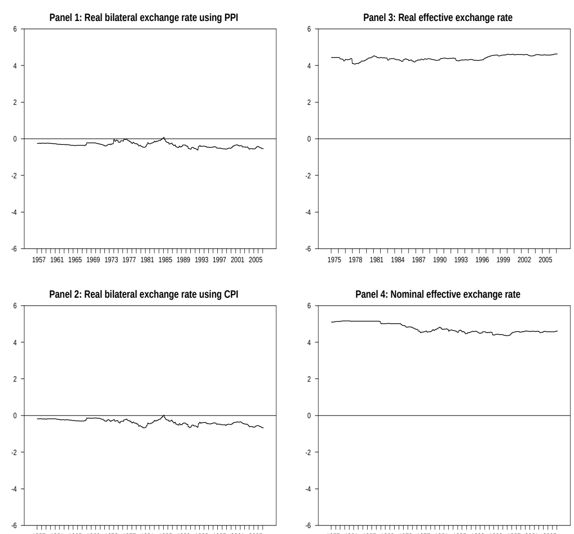
Sweden



Switzerland



UK



New Zealand