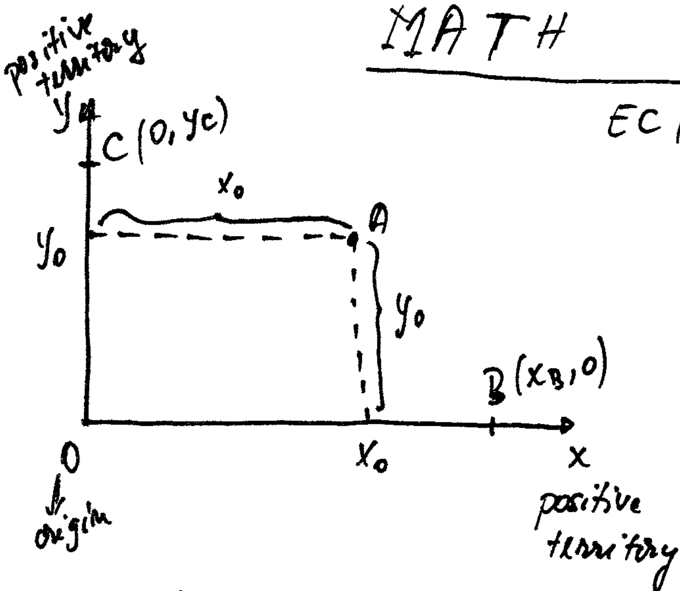
# MATH HANDOUT

6/5/2007

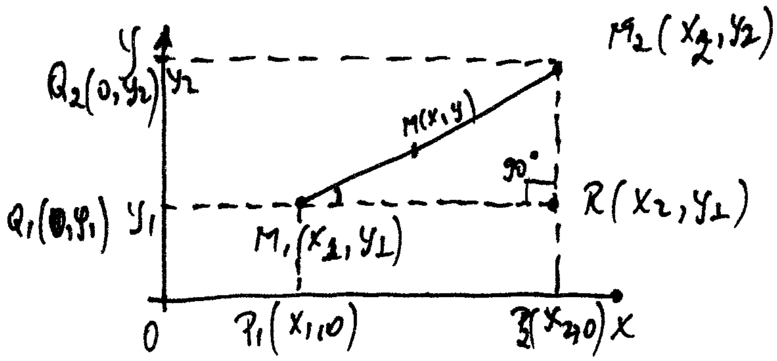
EC 110 - 001




We can draw any positive number, say $A(x_0, y_0)$ by specifying its coordinates on the horizontal axis (i.e., Ox) and on the vertical axis (i.e. Oy). The x-axis is sometimes called "abscissa", and the y-axis is sometimes called the "ordinate" axis.

A point $B(x_B, 0)$ belongs to the x-axis, and a point $C(0, y_C)$ belongs to the y-axis.

## Distance between two points




Assuming that the line formed by two points $M_1(x_1, y_1)$ and $M_2(x_2, y_2)$ is not parallel to either axis, we can compute the distance $M_1M_2$ in the following way: denote by $P_1$ and $Q_1$ and $P_2$, $Q_2$ the intersection points of the lines drawn from the points $M_1$ and $M_2$ orthogonal to the $Ox$ and $Oy$- axis, respectively. Let $R$ denote the intersection point of the lines $Q_1M_1$ and $P_2M_2$.

Now you should realize that the triangle $M_1RM_2$ is a right triangle. For any right triangle, the Pythagorean theorem states the square of the hypothenuse (i.e., the segment that opposes the right angle) is equal to the sum of the squares of the other two sides.

Thus: $M_1M_2^2 = M_1R^2 + M_2R^2$

But $M_1R = x_2 - x_1$

and $M_2R = y_2 - y_1$

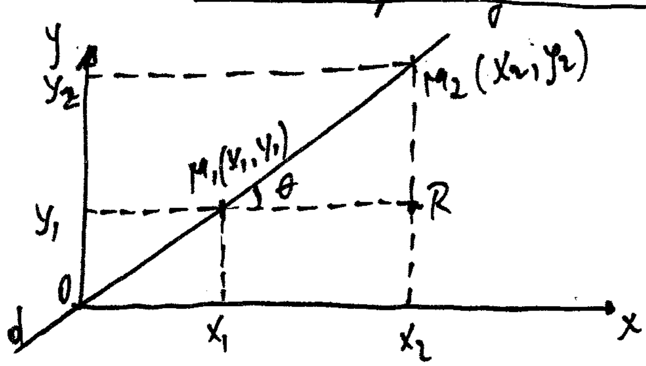
$$\Rightarrow M_1M_2^2 = (x_2-x_1)^2 + (y_2-y_1)^2 \Rightarrow \boxed{M_1M_2 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

Obs This distance formula is valid also for the special cases where $M_1M_2 \parallel Ox$ and $M_1M_2 \parallel Oy$ (i.e., the "$\parallel$" sign denotes being parallel to).

The midpoint $M(x, y)$ of $M_1M_2$ is given by:

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

## The slope of a curve (a line)




Denote by $\theta$ the measure in ~~~~ radians of the angle between the segments $M_1M_2$ and $M_1R$. ($0 \leq \theta \leq \pi$)

One radian (i.e. one unit) = $\frac{180}{\pi}$ degrees $\approx 57.3^\circ$

Now, the tangent of angle $\theta$ denoted by "$tg\,\theta$" is given by the ratio of $M_2R$ over $M_1R$ : $tg\,\theta = \frac{M_2R}{M_1R}$

<u>Definition</u> The real number $tg\,\theta$ is called the <u>slope</u> of line $d$ (where line $d$ is formed by extending the segment $M_1M_2$ at both extremities).

<u>Theorem</u> Let $m = tg\,\theta$ = slope of line $d$. The slope "$m$" of line $d$ is given by the formula:

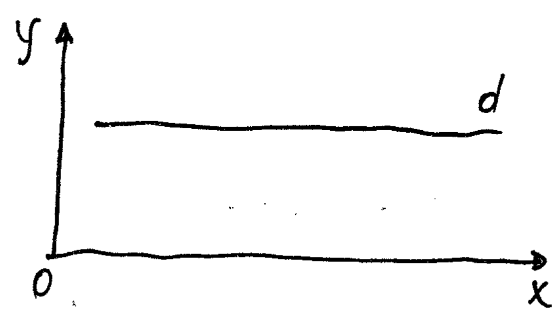
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \quad \left( = \frac{\Delta \text{Rise}}{\Delta \text{Run}} \right)$$

where $M_1(x_1, y_1)$ and $M_2(x_2, y_2)$ are any two points fixed on the line $d$.

<u>Proof.</u> $m = tg\,\theta = \frac{M_2R}{M_1R} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

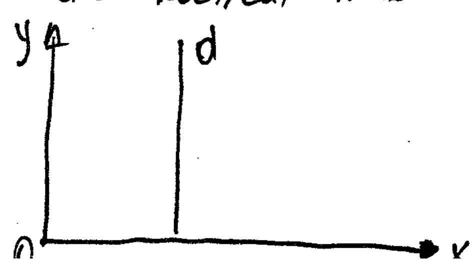
<u>Special cases</u>

1. $d$ = horizontal line




slope of $d = \frac{\Delta y}{\Delta x} \approx \frac{0}{\Delta x} = 0$

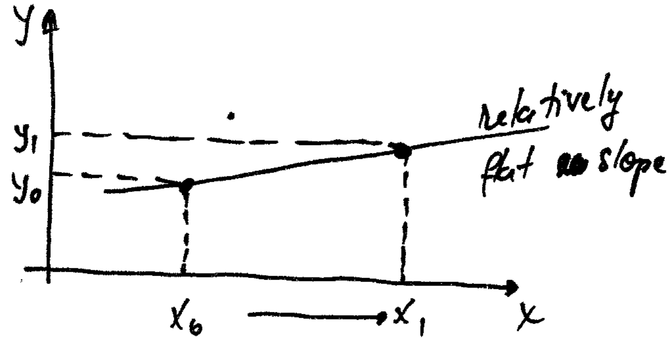
2. $d$ = vertical line




slope of $d = \frac{\Delta y}{\Delta x} \approx \frac{\Delta y}{0} \approx +\infty$ (infinity)

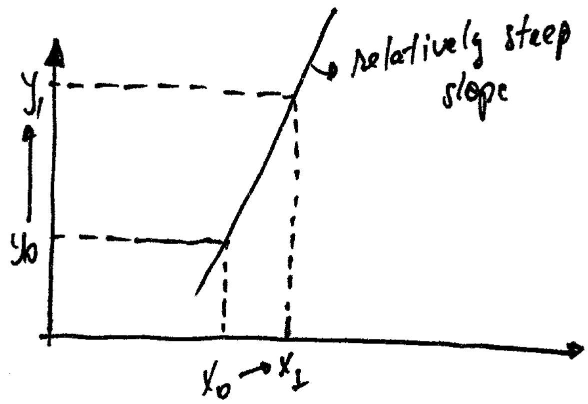
## Interpretation

(a)




: a big change in x causes a much smaller change in y (so slope here is a small number less than 1) : $\frac{\Delta y}{\Delta x} < 1$

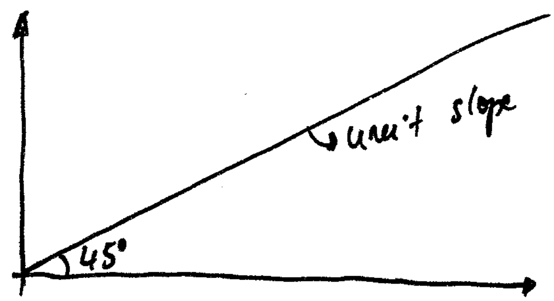
(b)




: a small change in x causes a much bigger change in y (so slope here is a number > 1)

slope $= \frac{\Delta y}{\Delta x} > 1$

(c)

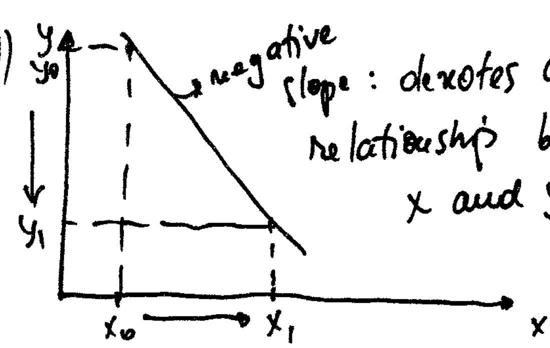



: here slope = 1

A change in x causes the same absolute change in y

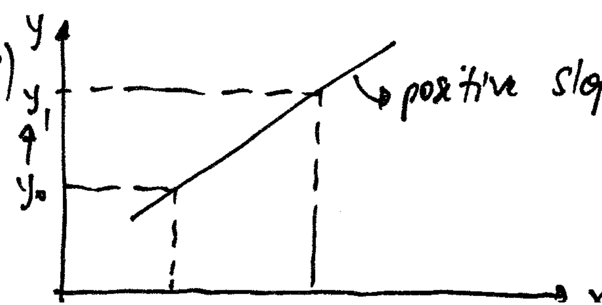
$$m = 1 = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0} \Rightarrow \Delta y = \Delta x$$

(d)




negative slope : denotes a negative relationship between x and y

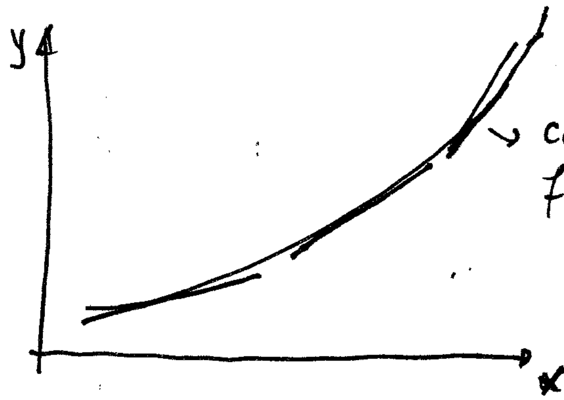
(e)




positive slope : denotes a positive relationship between x and y

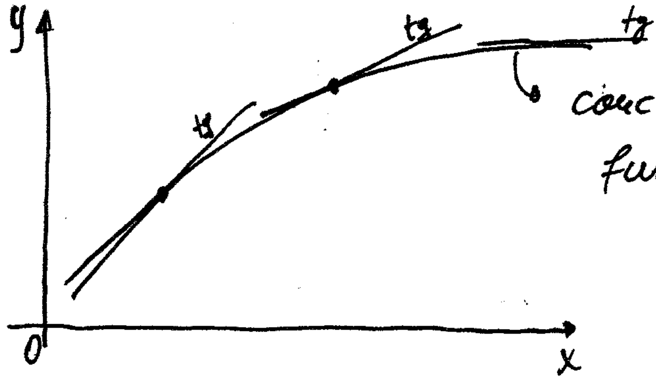
## Convex and Concave Curves

(1)




convex function : y increases at an increasing rate. At each point, the slope (given by the slope of the tangent to the curve at that point) becomes steeper.

(2)




concave function : y increases at a decreasing rate ; the slope ~~given by~~ ~~the~~ of the tangent at any point on the curve becomes flatter as we move to the right

Tips Convex : "holds water"

Concave : "does not hold water".