EXAM 3 Solutions

Intermediate Microeconomics EC 308-004
December 7, 2007 **Name:** ____________________
by writing my name i swear by the honor code

Read all of the following information before starting the Assignment:

- This is an individual take-home exam. I might take you off points if I see any hint of collusion.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- Answer any 5 of the 7 problems for 100 points overall. Do not answer to more than 5 problems. It is your responsibility to make sure that you have all of the answers!
- This exam will be due Thursday, December 6 in class.
- Good luck!

1. (*20 points*) PROBLEM 1: This problem has several distinct parts.

a. (*5 pts*) PART A: A monopolist faces a demand with constant elasticity of -3.0. It has a constant marginal cost of \$30 per unit and sets a price to maximize profit. If marginal cost should increase by 25%, would the price charged also rise by 25%?

We know:

$$\frac{P - MC}{P} = -\frac{1}{E_d} \tag{1}$$

or

$$P = \frac{MC}{1 + \frac{1}{E_d}} \tag{2}$$

Given that $E_d = -3.0$ it implies that $\frac{1}{E_d} = -\frac{1}{3}$ so that $P = \frac{3}{2}MC$. Thus, when $MC = 30$ then $P = 45$; when $MC = 37.5$ then $P = 56.25$. Thus P also increases by 25%. The answer is YES.

b. (*5 pts*) PART B: A firm faces the following average revenue (demand) curve: $P = 120 - 0.02Q$, where Q is weekly production and P is price, measured in cents per unit. Firm's cost function is given by $C = 60Q + 25,000$. Assume that the firm maximizes profits. What is the level of production, price, and total profit per week?

We have:

$$MR = 120 - 0.04Q \tag{3}$$

and $MC = 60$. Thus, at the optimal level $MC = MR$, which implies that $Q = 1,500$, $P = 120 - 0.002 \times 1,500 = 90$. Next,

$$\pi = TR - TC = P \times Q - C(Q) = \$200. \tag{4}$$

c. (*10 pts*) PART C: If the government decides to levy a tax of 14 cents per unit on this product, what will be the new level of production, price, and profit? (Hint: The new price becomes $P' = P + T$).

Method 1: Suppose that consumers pay the tax. The new price received by suppliers (i.e., monopolist) becomes:

$$P + T = 120 - 0.02Q \Rightarrow P = 120 - 0.02Q - T. \tag{5}$$

Next,

$$P + T = 120 - 0.04Q \Rightarrow P = 120 - 0.02Q - T. \tag{6}$$

such that the monopolist's revenue decrease by $T \times Q$, where $T = 14 cents$. Thus $P = 120 - 0.02 \times 1,150 - 14 = 83 cents$. Also,

$$\pi = 83 \times 1,150 - [60 \times 1,150 + 25,000] = 1450\text{cents} = 14.50\$/week. \tag{7}$$

Note, the price consumers pay is $P_B = 97 cents$ and the price the monopolist receives is $P_M = 83 cents$. Thus, consumers and the monopolist each pay 7 cents.

Method 2: if the monopolist paid the tax $\Rightarrow$

$$TC_M = 60Q + 25,000 + TQ = (60 + T)Q + 25,000. \tag{8}$$

This gives $MC = 60 + T$. Set $MC = MR$ to get $Q = 1,150$. The rest follows similarly.

2. (*20 points*) PROBLEM 2: A monopolist faces the demand curve $P = 11 - Q$, where P is measured in dollars per unit and Q in thousands of units. The monopolist has a constant average cost of \$6 per unit.

a. (*10 pts*) PART A: Draw the AR, MR, AC, and MC curves. What are the monopolist's profit-maximizing price and quantity? What is the resulting profit? Calculate the firm's degree of monopoly power using the Lerner index.

$$AR : P = 11 - Q \tag{9}$$
$$MR : P = 11 - 2Q \tag{10}$$
$$\text{AC=cst.} \Rightarrow MC = 6. \tag{11}$$

Set $MC = MR$ to get that $11 - 2Q = 6 \Rightarrow Q = 2.5$, $P = 8.5$, $\pi = \$6,250$.

Lerner index:

$$\frac{P - MC}{P} = \frac{8.5 - 6}{8.5} = 0.294. \tag{12}$$

b. (*5 pts*) PART B: A government regulatory agency sets a price ceiling of \$7 per unit. What quantity will be produced, and what will the firm's profit be? What happens to the degree of monopoly power?

$$7 = 11 - Q \Rightarrow Q = 4, \text{(or 4,000)}. \tag{13}$$

Since $7 > MC = 6 \Rightarrow$ positive monopoly profits. Then, $\pi = 7 \times 4,000 - 6 \times 4,000 = 4,000$.

Lerner index:

$$\frac{P - MC}{P} = \frac{7-6}{7} = 0.143. \tag{14}$$

c. (*5 pts*) PART C: What price ceiling yields the largest level of output? What is the firm's degree of monopoly power at this price?

When $P \approx MC \Rightarrow Q = Q_{max}$. So when $P < MC$, the monopolist goes out of business, and when $P > MC$ then $Q < Q_{max}$ and the industry is not competitive. Thus set $P = P + \xi$, where $\xi > 0$ but small. Then the Lerner index is:

$$\frac{P - MC}{P} = \frac{6 + \xi - 6}{6} = \frac{\xi}{6} \rightarrow 0, \text{as } \xi \rightarrow 0. \tag{15}$$

3. (*20 points*) PROBLEM 3: (Chapter 11 problem) Suppose that BMW can produce any quantity of cars at a constant MC equal to \$20,000 and fixed cost of \$10 billion. You are asked to advise the CEO as to what prices and quantities BMW should set for sales in Europe and in the US. The demand for BMWs in each market is given by:

$$Q_E = 4,000,000 - 100P_E \qquad (16)$$
$$Q_{US} = 1,000,000 - 20P_{US} \qquad (17)$$

Assume that BMW can restrict US sales to authorized BMW dealers only.

a. (*10 pts*) PART A: What quantity of BMWs should the firm sell in each market, and what will the price be in each market? What will total profit be?

BMW maximizes π over the quantities produced. Thus, write:

$$P_E = 40,000 - \frac{Q_E}{100} \qquad (18)$$
$$P_{US} = 50,000 - \frac{Q_{US}}{20} \qquad (19)$$

Next, write:

$$\pi = TR - TC = P_E Q_E + P_{US} Q_{US} - 20,000(Q_E + Q_{US}) - 10,000,000,000, \qquad (20)$$

$$\pi = \left(40,000 - \frac{Q_E}{100}\right) QE + \left(\frac{50,000 - Q_{US}}{20}\right) Q_{US} - 20,000(Q_E + Q_{US}) - 10,000,000,000. \qquad (21)$$

Take FOC w.r.t. Q_E and Q_{US}:

$$\frac{\partial \pi}{\partial Q_E} = 40,000 - \frac{Q_E}{50} - 20,000 = 0 \Rightarrow Q_E = 1,000,000 \Rightarrow P_E = 30,000. \qquad (22)$$
$$\frac{\partial \pi}{\partial Q_{US}} = 50,000 - \frac{Q_{US}}{10} - 20,000 = 0 \Rightarrow Q_E = 300,000 \Rightarrow P_{US} = 35,000. \qquad (23)$$

Next, $\pi = \$4.5 bill$.

b. (*10 pts*) PART B: If BMW were forced to charge the same price in each market, what would be the quantity sol in each market, the equilibrium price, and the company's profit?

We have that $Q_T = Q_E + Q_{US} = 5,000,000 - 120P \Rightarrow P = \frac{5,000,000}{120} - \frac{Q}{120}$.Thus,

$$MR = \frac{5,000,000}{120} - \frac{Q}{60} \qquad (24)$$

Set $MC = MR$ to find that $Q = 1,300,000$ and $P = 30833.33$. Plug this value in the expressions above to find that $Q_E = 916,667$ and $Q_{US} = 383,333$. Finally, $\pi = 4,083,333,330$.

4. (*20 points*) PROBLEM 4: This question has 2 parts.

a. (*10 pts*) PART A: Many retail video stores offer two alternative plans for renting films:

- A two-part tariff: Pay an annual membership fee (e.g., $20) and then pay a small fee for the daily rental of each film (e.g., $2 per film per day).
- A straight rental fee: Pay no membership fee, but pay a higher rental fee (e.g., $4 per film per day).

What is the logic behind the two-part tariff in this case? Why offer the customer a choice of two plans rather than simply a two-part tariff?

By employing this strategy, the firm allows consumers to sort themselves into two groups, or markets (assuming that subscribers do not rent to non-subscribers): high-volume consumers who rent many movies per year (here, more than 20) and low-volume consumers who rent who rent only a few movies per year (less than 20). If only a two-part tariff is offered, the firm has the problem of determining the profit-maximizing entry and rental fees with many different consumers. A high entry fee with a low rental fee discourages low-volume consumers from subscribing. A low entry fee with a high rental fee encourages membership, but discourages high-volume customers from renting. Instead of forcing customers to pay both an entry and rental fee, the firm effectively charges two different prices to two types of consumers.

b. (*10 pts*) PART B: In June/July 2007, Apple launched the iPhone. However, only two months after the event, Apple slashed the price by $200. Given the discussion in Chapter 11, why do you think Apple make this abrupt move?

Razvan's thoughts:
By employing this strategy, Apple allowed consumers to sort themselves into two groups, or markets: die-hard Apple fans who know the quality of Apple products and are willing to pay the high price; potential future Apple fans who need some sort of incentive (i.e., lower prices) to join the fan base. Apple probably approximated that two months were enough to lure all hard-core Apple fans. Next, it decided to lure future more fans by slashing prices and allowing more cosumers to enjoy the quality of its products.

5. (*20 points*) PROBLEM 5: Suppose that two identical firms produce widgets and that they are the only firms in the market. Their costs are given by $C_1 = 60Q_1$ and $C_2 = 60Q_2$, where Q_1 is the output of firm 1 and Q_2 is the output of firm 2. Price is determined by the following demand curve: $P = 300 - Q$, where $Q = Q_1 + Q_2$.

a. (*10 pts*) PART A: Find the Cournot-Nash equilibrium. Calculate the profit of each firm at this equilibrium.

Write:

$$\pi_1 = PQ_1 - C_1 = (300 - Q_1 - Q_2)Q_1 - 60Q_1 = 300Q_1 - Q_1^2 - Q_1Q_2 - 60Q_1 \quad (25)$$

$$\pi_2 = PQ_2 - C_2 = (300 - Q_1 - Q_2)Q_2 - 60Q_2 = 300Q_2 - Q_1Q_2 - Q_2^2 - 60Q_2 \quad (26)$$

Take the FOCs:

$$\frac{\partial \pi}{\partial Q_1} = 300 - 2Q_1 - Q_2 = 0 \Rightarrow Q_1 = 120 - 0.5Q_2 \quad (27)$$

$$\frac{\partial \pi}{\partial Q_2} = 300 - Q_1 - 2Q_2 = 0 \Rightarrow Q_2 = 120 - 0.5Q_1 \quad (28)$$

Thus:

$$Q_1 = 120 - 0.5[120 - 0.5Q_1] = 60 - 0.25Q_1 \Rightarrow Q_1 = 80 \quad (29)$$

Similarly find $Q_2 = 80$ such that $\pi_1 = \pi_2 = 6,400$.

b. (*5 pts*) PART B: Suppose the two firms form a cartel to maximize joint profits. How many widgets will be produced? Calculate each firm's profit. (hint: Assume $Q = Q_1 = Q_2$).

Method 1: The two firms act as a monopolist, where each firm produces an equal share of total output. Demand is given by $P = 300 - Q$, $MR = 300 - 2Q$, and $MC = 60$. Set $MC = MR$ to find that $Q = 120$ and $Q_1 = Q_2 = 60$, respectively. Therefore:

$$\pi_1 = \pi_2 = 180 \times 60 - 60 \times 60 = 7,200. \tag{30}$$

Method 2: The firms agree to produce $Q = Q_1 = Q_2$ so that the market price is given by $P = 300 - 2Q$. Now:

$$\pi_1 = \pi_2 = (300 - 2Q)Q - 60Q = 300Q - 2Q^2 - 60Q \tag{31}$$

Take the FOC to maximize profit to find that $300 - 4Q - 60 = 0$. This solves for $Q = Q_1 = Q_2 = 60$ and $P = 300 - 2 \times 60 = 180$. Finally, find that $\pi_1 = \pi_2 = 7,200$.

c. (*5 pts*) PART C: Suppose firm 1 abides by the agreement, but firm 2 cheats by producing its profit maximizing level. How many widgets will firm 2 produce? What will be each firm's profits?

Firm 2 knows that $Q_1 = 60$ and given the reaction function derived in part (a) firm 2 sets $Q_2 = 120 - 0.5 \times 60 = 90$. Overall, $Q_T = 150$ and $P = 300 - 150 = 150$. Hence:

$$\pi_1 = 150 \times 60 - 60 \times 60 = 5,400 \tag{32}$$
$$\pi_2 = 150 \times 90 - 60 \times 90 = 8,100. \tag{33}$$

By cheating, firm 2 increases its profits at firm 1's expense.

6. (*20 points*) PROBLEM 6: Demand for light-bulbs can be characterized by $Q = 100 - P$ where Q is in millions of lights sold, and P is the price per box. There are two producers of lights: Everglow and Dimlit. They have identical cost functions: $C_i = 10Q_i + \frac{1}{2}Q_i^2$ where $i = E, D$, $Q = Q_E + Q_D$.

a. (*10 pts*) PART A: Unable to recognize the potential for collusion, the two firms act as short-run perfect competitors. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

Method 1:

$$Q = 100 - P \Rightarrow P = 100 - Q_E - Q_D \tag{34}$$

Next find that $MC = 10 + Q_i$. In a competitive industry we know that $P = MC_i$. This leads to a system of 2 equations with 2 unknowns:

$$\begin{aligned} 100 - Q_E - Q_D &= 10 + Q_E \Leftrightarrow 2Q_E + Q_D = 90 &(35)\\ 100 - Q_E - Q_D &= 10 + Q_D \Leftrightarrow Q_E + 2Q_D = 90 &(36)\end{aligned}$$

which solves for $Q_E = Q_D = 30$ and $P = 40$. Finally, $\pi_E = \pi_D = 40 \times 30 - 10 \times 30 - \frac{1}{2}30^2 = 450$.

Method: one can alternatively find the equilibrium quantity and price by equating the market supply and demand. Market supply is given by the sum of the MCs. Thus, $MC = P = 10 + Q_i \Rightarrow Q_E + Q_D = 2P - 20$. Thus:

$$MS = MD \Leftrightarrow 2P - 20 = 100 - P \Leftrightarrow 3P = 120 \Rightarrow P = 40 \tag{37}$$

The rest follows as above.

b. (*10 pts*) PART B: Top management in both firms is replaced. Each new manager independently recognizes the oligopolistic nature of the light bulb industry and plays Cournot. What are the equilibrium values of Q_E, Q_D, and P? What are each firm's profits?

Write:

$$\begin{aligned} \pi_E &= PQ_E - C_E = (100 - Q_E - Q_D)Q_E - 10Q_E - 0.5Q_E^2 &(38)\\ \pi_D &= PQ_D - C_D = (100 - Q_E - Q_D)Q_D - 10Q_D - 0.5Q_D^2 &(39)\end{aligned}$$

Take the FOCs:

$$\begin{aligned} \frac{\partial \pi}{\partial Q_E} &= 90 - 3Q_E - Q_D = 0 \Rightarrow Q_E = \frac{90 - Q_D}{3} &(40)\\ \frac{\partial \pi}{\partial Q_D} &= 90 - 3Q_D - Q_E = 0 \Rightarrow Q_D = \frac{90 - Q_E}{3} &(41)\end{aligned}$$

Thus:

$$Q_E = 30 - \frac{1}{3}\frac{90 - Q_E}{3} \Rightarrow Q_E = 22.5 \tag{42}$$

Similarly find $Q_D = 22.5$ such that $\pi_E = \pi_D = 759.38$.

7. (*20 points*) PROBLEM 7: This problem has 2 parts:

a. (*10 pts*) PART A: You are offered the choice of two payment streams: (a) \$150 paid one year from now and \$150 paid two years from now; (b) \$130 paid one year from now and \$160 paid two years from now. Which payment stream would you prefer in the interest rate is 5%? If it is 10%?

$$PDV = \frac{\$150}{(1+0.05)} + \frac{\$150}{(1+0.05)^2} = 278.91 \tag{43}$$

$$PDV = \frac{\$130}{(1+0.05)} + \frac{\$160}{(1+0.05)^2} = 268.93 \tag{44}$$

$$PDV = \frac{\$150}{(1+0.1)} + \frac{\$150}{(1+0.1)^2} = 260.33 \tag{45}$$

$$PDV = \frac{\$130}{(1+0.1)} + \frac{\$160}{(1+0.1)^2} = 250.41 \tag{46}$$

In both cases choose stream 1.

b. (*10 pts*) PART B: A bond has two years to mature. It makes coupon payments of \$100 after one year and both a coupon payment of \$100 and a principal payment of \$1000 after two years. The bond is selling for \$966. What is its yield rate? (Hint: use quadratic formula. If $ax^2 + bx + c = 0$ then $x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$)

$$966 = \frac{\$100}{(1+R)} + \frac{\$1100}{(1+R)^2} \tag{47}$$

Multiply across by $(1+R)^2$ to get $966R^2 + 1,832R - 234 = 0$. Using the quadratic formula find that $R = 12\%$ (the other value is negative).

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