EXAM 2 SOLUTIONS

Intermediate Microeconomics EC 308-004
November 20, 2007 **Name:** ______________________
by writing my name i swear by the honor code

Read all of the following information before starting the Assignment:

- This is an individual exam.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- Answer any 5 of the 7 problems for 100 points overall. Do not answer to more than 5 problems. It is your responsibility to make sure that you have all of the answers!
- Note that there is a 2-point bonus Survey Question at the end of the exam.
- Good luck!

1. (*20 points*) PROBLEM 1: Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant? (Hint: For the second question take the first derivative of the MP with respect to that factor).

a. (*5 pts*) PART A: $q(K, L) = 3LK + 2K$

Use numerical examples. For instance, whenever one doubles the inputs and the output doubles, then a function displays CRS. If the output more than doubles when you double the inputs, then the function displays increasing returns to scale (IRS for short). On the contrary, if the output less than doubles, then the function has decreasing returns to scale (DRS for short). Let's use $K = 2$ and $L = 2$. Then: $q(2, 2) = 16$. When $K = 4$ and $L = 4$, then $q(4, 4) = 56$. Since q more than doubles when we double K and L, then this function displays IRS.
Second question actually asks to verify whether the MP_K and MP_L are constant, increasing or decreasing. Mathematically, it requires us to take the first derivative of the MP with respect to the inputs (or difference the function q twice). First, we have:

$$MP_K = \frac{\partial q(K, L)}{\partial K} = 3L + 2 \tag{1}$$

and

$$MP_L = \frac{\partial q(K, L)}{\partial L} = 3K. \tag{2}$$

Next, it is easy to see that the MPs are constant (i.e., $\frac{\partial MP_K}{\partial K} = 0$ and $\frac{\partial MP_L}{\partial L} = 0$).

b. (*5 pts*) PART B: $q(K, L) = (4L + 4K)^{1/2}$

For all $t > 1$ we have:

$$q(tK, tL) = \sqrt{4tL + 4tK} = \sqrt{t}\sqrt{4L + 4K} < t\sqrt{4L + 4K} = tq(K, L) \tag{3}$$

which implies that the function displays DRS. Alternatively, one can verify numerically that

$$q(4, 4) = \sqrt{32} = 5.66 < 2 \times q(2, 2) = 2 \times 4 = 8. \tag{4}$$

Next:

$$MP_K = \frac{\partial q(K, L)}{\partial K} = \frac{2}{\sqrt{4L + 4K}} \tag{5}$$

$$MP_L = \frac{\partial q(K, L)}{\partial L} = \frac{2}{\sqrt{4L + 4K}} \tag{6}$$

and one can see that:

$$\frac{\partial MP_K}{\partial K} = \frac{\partial^2 q(K, L)}{\partial K^2} = -\frac{4}{\sqrt{(4L + 4K)^3}} < 0 \tag{7}$$

$$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 q(K, L)}{\partial L^2} = -\frac{4}{\sqrt{(4L + 4K)^3}} < 0 \tag{8}$$

Thus, both MPs are decreasing.

c. **(***5 pts***)** PART C: $q(K,L) = ln(L^2K)$

Given that:

$$q(4,4) = ln(64) = 4.16 = 2 \times q(2,2) \tag{9}$$

the function displays CRS.
Next:

$$MP_K = \frac{\partial q(K,L)}{\partial K} = \frac{1}{K} \tag{10}$$

$$MP_L = \frac{\partial q(K,L)}{\partial L} = \frac{2}{L} \tag{11}$$

and one can see that:

$$\frac{\partial MP_K}{\partial K} = \frac{\partial^2 q(K,L)}{\partial K^2} = -\frac{1}{K^2} < 0 \tag{12}$$

$$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 q(K,L)}{\partial L^2} = -\frac{2}{L^2} < 0 \tag{13}$$

Thus, both MPs are decreasing.

d. **(***5 pts***)** PART D: $q(K,L) = L^{1/3}K^{2/3}$

For all $t > 1$ we have:

$$q(tK,tL) = (tL)^{1/3}(tK)^{2/3} = tL^{1/3}K^{2/3} = tq(K,L) \tag{14}$$

In this case, the function displays CRS.
Next:

$$MP_K = \frac{\partial q(K,L)}{\partial K} = \frac{2}{3}\left(\frac{L}{K}\right)^{1/3} \tag{15}$$

$$MP_L = \frac{\partial q(K,L)}{\partial L} = \frac{1}{3}\left(\frac{K}{L}\right)^{2/3} \tag{16}$$

and one can see that:

$$\frac{\partial MP_K}{\partial K} = -\frac{2}{9}\left(\frac{L}{K^4}\right)^{1/3} < 0 \tag{17}$$

$$\frac{\partial MP_L}{\partial L} = -\frac{2}{9}\left(\frac{K^2}{L^5}\right)^{1/3} < 0 \tag{18}$$

Thus, both MPs are decreasing.

2. (*20 points*) PROBLEM 2: The production function for a product is given by $q(K, L) = 10\sqrt{K} + 5\sqrt{L}$. If the price of capital is \$12 per day and the price of labor \$3 per day, what is the minimum cost of producing 240 units of output? (Hint: Set-up the Lagrangian to minimize $C = wL + rK$ subject to the constraint that $q(K, L) = 240$).

Objective: minimize $C = wL + rK$ such that $q(K, L) = 240$.

Step 1. Write the Lagrangian:

$$L = 3L + 12K - \lambda[10\sqrt{K} + 5\sqrt{L} - 240] \tag{19}$$

Step 2. Take the FOCs:

$$\frac{\partial L}{\partial L} = 3 - \lambda\frac{5}{2\sqrt{L}} = 0 \Rightarrow \lambda = \frac{6\sqrt{L}}{5} \tag{20}$$

$$\frac{\partial L}{\partial K} = 12 - \lambda\frac{10}{2\sqrt{K}} = 0 \Rightarrow \lambda = \frac{24\sqrt{K}}{10} \tag{21}$$

$$\frac{\partial L}{\partial \lambda} = 10\sqrt{K} + 5\sqrt{L} - 240 = 0 \tag{22}$$

$$\tag{23}$$

Step 3. From the first two equations find that $L = 4K$. Plug this in the last equation to find $10\sqrt{K} + 5\sqrt{4K} = 240$ which solves for $K = 144$, and $L = 576$.

3. (*20 points*) PROBLEM 3: Suppose a production function is given by $F(K, L) = K^2L$; the price of capital is \$10 and the price of labor is \$15. What combination of labor and capital maximizes the output given a fixed cost of 85? (Hint: Set-up the Lagrangian to maximize $q(K, L)$ subject to the constraint that $wL + rK = 85$).

Objective: maximize $q(K, L)$ subject to $wL + rK = 85$.

Step 1. Write the Lagrangian:

$$L = K^2L - \lambda[15L + 10K - 85] \tag{24}$$

Step 2. Take the FOCs:

$$\frac{\partial L}{\partial K} = 2KL - 10\lambda = 0 \Rightarrow \lambda = \frac{2KL}{10} \tag{25}$$

$$\frac{\partial L}{\partial L} = K^2 - 15\lambda = 0 \Rightarrow \lambda = \frac{K^2}{15} \tag{26}$$

$$\frac{\partial L}{\partial \lambda} = 15L + 10K - 85 = 0 \tag{27}$$

$$\tag{28}$$

Step 3. From the first two equations find that $K = 3L$. Plug this in the last equation to find that:

$$15L + 30L - 85 = 0 \Rightarrow 45L = 85 \tag{29}$$

Thus $L = 1.89$ and $K = 5.67$.

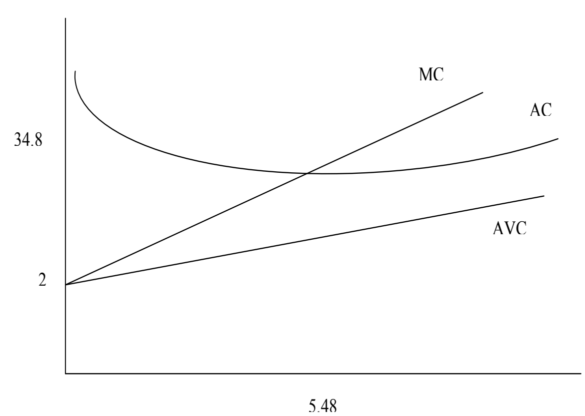
4. (*20 points*) PROBLEM 4: This question has 4 parts. Suppose that a firm's cost function is $C(q) = 3q^2 + 2q + 90$.

a. (*5 pts*) PART A: Find variable cost, fixed cost, average cost, average variable cost, average fixed cost, and marginal cost.

$$VC = 3q^2 + 2q \tag{30}$$
$$FC = 90 \tag{31}$$
$$AC = \frac{C(q)}{q} = 3q + 2 + \frac{90}{q} \tag{32}$$
$$AVC = \frac{VC}{q} = 3q + 2 \tag{33}$$
$$AFC = \frac{FC}{q} = \frac{90}{q} \tag{34}$$
$$MC = 6q + 2 \tag{35}$$

b. (*5 pts*) PART B: Show the average cost, marginal cost, and average variable cost curves on a graph.

AC is u-shaped. AC is relatively large at first because the firm is not able to spread the fixed cost over many units of output. As output increases, AFC will fall relatively rapidly. AC will increase at some point because the AFC will become very small and AVC is increasing as q increases. AVC will increase because of diminishing returns to the variable factor labor. MC and AVC are linear, and both pass through the origin. AVC is everywhere below AC. MC is everywhere above AVC. If the average is rising, then the marginal must be above the average. MC will hit the AC at its minimum point.



c. (*5 pts*) PART C: Find the output that minimizes average cost. The minimum AC quantity is where $MC = AC$:

$$AC = 3q + 2 + \frac{90}{q} = 6q + 2 = MC \Rightarrow \frac{90}{q} = 3q \Rightarrow 30 = q^2 \Rightarrow q = 5.48. \tag{36}$$

d. (*5 pts*) PART D: At what range of prices will the firm earn a positive profit? How about a negative profit?

The firm will earn negative profit when $P = MC < AC$, or at any price below minimum AC. In part C above we found that the minimum AC quantity was $q = 5.48$. Plug $q = 5.48$ into the average cost function to find $AC = 34.88$. The firm will therefore earn negative profit if price is below 34.88 and positive otherwise.

5. (*20 points*) PROBLEM 5: This problem has 3 parts.

a. (*10 pts*) PART A: Suppose that a firm's production function is $q = \sqrt{1+4x}$ in the short run, where there are fixed costs of \$1000, and x is the variable input whose cost is \$4,000 per unit. What is the total cost of producing a level of output q? In other words, identify the total cost function $C(q)$.

The total cost function is:

$$C(x) = FC + VC = 1000 + 4000x. \tag{37}$$

Now rewrite the production function to express x in terms of q so that $x = \frac{q^2-1}{4}$. We can then substitute this into the above cost function to find $C(q)$:

$$C(q) = 4000 \times \frac{q^2-1}{4} + 1000 = 1000q^2 - 1000 + 1000 = 1000q^2. \tag{38}$$

b. (*5 pts*) PART B: Write down the equation for the supply curve.

The firm supplies output where $P = MC$ so the marginal cost curve is the supply curve, or $P = 2000q$.

c. (*5 pts*) PART C: If the price is \$6000, how many units will the firm produce? What is the level of profit? Illustrate your answer on a cost-curve graph.

To figure this out, set price equal to marginal cost to find:

$$P = 2000q = 6000 \Rightarrow q = 3. \tag{39}$$

Profit is $\pi = 6000 \times 3 - 1000 \times 9 = 9000$. Graphically, the firm produces where the price line hits the MC curve. Since profit is positive, this will occur at a quantity where the price is greater than AC. To find profit on the graph, take the difference of the revenue box ($P \times q$) and the cost box ($AC \times q$).

6. (*20 points*) PROBLEM 6: Among the tax proposals regularly considered by Congress is an additional tax on distilled liquors. The tax would not apply to Romanian vodka. The price elasticity of supply of liquor is 3.0, and the price elasticity of demand is -0.2.
The cross price elasticity of demand for mixing drinks with respect to the price of liquor is -0.1. Also, the cross price elasticity of demand for Romanian vodka with respect to the price of liquor is 0.1.

a. (*10 pts*) PART A: If the new tax is imposed, who will bear the greater burden - liquor suppliers or liquor consumers? Why?

Consumers will bear:

$$\frac{E_S}{E_S - E_D} = \frac{3}{3-(-0.2)} = \frac{2}{2.5} = 0.94 \quad (40)$$

that is, 94% of the tax burden. This is true because supply is relatively elastic and demand is relatively inelastic.

Sellers support:

$$\frac{-E_B}{E_S - E_D} = \frac{-0.2}{3-(-0.2)} = \frac{-0.2}{3.2} = 0.06 \quad (41)$$

which represents 6% of the tax burden.

b. (*10 pts*) PART B: Assuming that mixing drinks supply is infinitely elastic, how will the new tax affect the mixing drinks market? Also, assuming that Romanian vodka supply is infinitely elastic, how will the new tax affect the Romanian vodka market?

Given that a 1% increase in the price of liquor leads to a 0.1% decrease in the demand for mixing drinks, the demand for the latter will shift to the left. However with a flat supply curve, the price of mixing drinks remains unchanged.
Given that a 1% increase in the price of liquor leads to a 0.1% increase in the demand for Romanian vodka, the demand for the latter will shift to the right. However with a flat supply curve, the price of mixing drinks remains unchanged.

7. (*20 points*) PROBLEM 7: Shortly discuss the following:

a. (*5 pts*) PART A: Economic Cost versus Accounting Cost

b. (*5 pts*) PART B: Opportunity Cost and Sunk Costs

c. (*5 pts*) PART C: Relationship between MRTS, prices, and input ratios at the equilibrium level.

$$MRTS = \frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} \tag{42}$$

Thus, the marginal rate of technical substitution between two inputs is equal to the ratio of the marginal products of inputs.

d. (*5 pts*) PART D: Market failure: externalities and lack of information.

Survey Question (2 Extra Credit Points):

How would you change/improve the material studied in this class? What would you suggest the instructor of this class (that's me) do to increase the quality of the course? Any other comments, suggestions, ideas?

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