EXAM 1 - SOLUTIONS

Intermediate Microeconomics EC 308-004
October 30, 2007 **Name:** ______________________________
by writing my name i swear by the honor code

Read all of the following information before starting the Exam:

- This is an individual exam.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This assignment has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the answers!
- Good luck!

1. (*20 points*) PROBLEM 1: This question has 3 parts:

a. (*5 pts*) PART A: For each city across the U.S., economists construct a price index for a similar basket of goods. In Tuscaloosa the index is 120.4 and in Birmingham the index is 89.8. If you have been offered $105,000 for a job in Tuscaloosa and $98,000 for a similar job in Birmingham, which job affords you the highest purchasing power of the bundle of goods in the price index? Use the Tuscaloosa value as the base year.

$$RW_{Birmingham} = \frac{120.4}{89.8}98,000 = \$131,394.2 \quad (1)$$

in Tuscaloosa dollars. Thus, choose the job in Birmingham.

b. (*5 pts*) Part B: Define the Bandwagon and Snob effects, respectively.

Bandwagon Effect: positive network externality in which the quantity of a good that an individual demands grows in response to the growth of purchases by other individuals.
Snob Effect: a negative network externality in which the quantity demanded falls in response to growth of purchases by other individuals.

c. (*10 pts*) PART C: The demand for Romanian sausages is estimated from this theoretical model:

$$Q = kP^a I^b A^c e \tag{2}$$

where Q = units per day, P = price per unit, A = advertising budget per month by sellers, I = per capita income of consumers, and e = random error. In a recent study, one researcher estimated the log-linear form of this equation with regression analysis as:

$$Q = 2.5 - 0.33logP + 0.15logI + 0.2logA \tag{3}$$

Explain what the coefficients of $logP$, $logI$, and $logA$ reveal about this product.

The coefficients of the variables are the respective elasticities of demand. The price elasticity is -0.33, income elasticity is 0.15, and the advertising elasticity is 0.2. These coefficients indicate that Romanian sausages are relatively price inelastic, are a normal good, and are responsive to advertising by Romanian marketers.

2. (*20 points*) PROBLEM 2: In a city with a medium sized population, the equilibrium price for a city bus ticket is \$2.00 and the number of riders each day is 100,000. The short-run price elasticity of demand is -0.50, and the short-run elasticity of supply is 1.2.

a. (*10 pts*) PART A: Estimate the short-run linear supply and demand curves for bus tickets.

We need to estimate $Q^D = a + bP$ and $Q^S = c + dP$, respectively.

We have E_P^D = -0.5 = $\frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{2}{100{,}000}b$. This implies that $b = -25,000$. Similarly one has that $E_P^S = 1.2 = \frac{P}{Q}\frac{\Delta Q}{\Delta P} = \frac{2}{100{,}000}d$ which implies that $d = 60,000$.

Now:

$$100,000 = a - 25,000 \times 2 \tag{4}$$

which gives a = 150,000. Similarly:

$$100,000 = c + 60,000 \times 2 \tag{5}$$

that gives c = -20,000.

b. (*5 pts*) PART B: If the demand for bus tickets increased by 10% because of a rise in the world price of oil, what would be the new equilibrium price of bus tickets?

New demand will be: Q^D = 1.1 × (150,000 - 60,000 P) = 165,000 - 27,500 P.
Now supply stays the same at Q^S = -20,000 + 60,000 P. Solve for the equilibrium to find that P = \$2.11.

c. (*5 pts*) PART C: If the city council refused to let the bus company raise the price of bus tickets after the demand for tickets increases (see (B) above), what daily shortage of tickets would be created?

Shortage = (165,000 - 27,500 × 2.00) - (-20,000 + 60,000 × 2) = 110,000 - 100,000 = 10,000.

3. (*20 points*)
PROBLEM 3: Suppose you are in charge of a toll bridge that costs essentially nothing to operate. The demand for bridge crossings Q is given by

$$P = 15 - \frac{1}{2}Q. \tag{6}$$

a. (*5 pts*) PART A: Draw the demand curve for bridge crossings. (Note: Clearly mark the intersection points with the horizontal and vertical axes, respectively).

Answer: the demand curve is linear and downward sloping. The vertical intercept is 15 and the horizontal intercept is 30.

b. (*5 pts*) PART B: How many people would cross the bridge if there were no toll?

Answer: Set $P = 15 - \frac{1}{2}Q = 0$ and solve for $Q = 30$.

c. (*5 pts*) PART C: What is the loss of consumer surplus (CS) associated with a bridge toll of \$5?

Answer: if the toll is $T = P = 5$ then $Q^D = 20$. The lost CS is the area below the price line of \$5 and to the left of the demand curve. That is $CS = 5 \times 20 + 0.5 \times (5 \times 10) = \$125$.

d. (*5 pts*) PART D: The toll bridge operator is considering an increase in the toll to \$7. At this new higher price, how many people would cross the bridge? Would the toll bridge revenue increase or decrease? What does your answer tell you about the elasticity of demand?

Answer: When $T = \$7$, then $Q^D = 16$. The initial toll revenue was $5 \times 20 = \$100$. The new toll revenue is $7 \times 16 = \$112$. Since the revenue went up when the toll was increased, demand is inelastic - the increase in price of 40% outweighed the decline in Q^D of 20%.

4. (*20 points*) PROBLEM 4: Bruno has the following utility function:

$$U(X,Y) = 40X + 80Y - lnX - 2lnY \tag{7}$$

where X is his consumption of CDs with a price of \$1 and Y is his consumption of movie videos, with a rental price of \$2. He plans to spend \$51 on both forms of entertainment.

a. (*10 pts*) PART A: Fine Bruno's marginal utility with respect to X and with respect to Y.

$$\frac{\partial L}{\partial X} = 40 - \frac{1}{X} \tag{8}$$

$$\frac{\partial L}{\partial Y} = 80 - \frac{2}{Y} \tag{9}$$

b. (*10 pts*) PART B: Determine the number of CDs and video rentals that will maximize Bruno's utility. What is his utility at these levels? (Hint: Use the Lagrangian method)

Step 1: Write out the Lagrangian.

$$L = 40X + 80Y - lnX - lnY - \lambda(X + 2Y - 51) \tag{10}$$

Step 2. Take the first order conditions (FOCs):

$$\frac{\partial L}{\partial X} = 40 - \frac{1}{X} - \lambda = 0 \tag{11}$$

$$\frac{\partial L}{\partial Y} = 80 - \frac{2}{Y} - 2\lambda = 0 \tag{12}$$

$$\frac{\partial L}{\partial \lambda} = X + 2Y - 51 = 0 \tag{13}$$

Step 3. Find the expressions for λ from (11) and (12), set them equal to each other and establish that X=Y. Next substitute X for Y (or Y for X) in (13) and solve to find that X=Y=17.

5. (*20 points*) PROBLEM 4: Suppose two investments have the same two payoffs, but the probability associated with each payoff differs, as illustrated in the table below:

Payoff	Probability (Investment A)	Probability(Investment B)
\$150	0.20	0.30
\$200	0.60	0.40
\$250	0.20	0.30

a. (*10 pts*) PART A: Find the expected return and standard deviation of each investment.

$$E(r_A) = 0.20 \times 150 + 0.60 \times 200 + 0.20 \times 250 = \$200 \quad (14)$$
$$\sigma_A = \sqrt{0.20 \times (150-200)^2 + 0.60 \times (200-250)^2 + 0.20 \times (250-200)^2} = \$31.62 \quad (15)$$

$$E(r_B) = 0.30 \times 150 + 0.40 \times 200 + 0.30 \times 250 = \$200 \quad (16)$$
$$\sigma_B = \sqrt{0.30 \times (150-200)^2 + 0.40 \times (200-200)^2 + 0.30 \times (250-200)^2} = \$38.73 \quad (17)$$

b. (*10 pts*) PART B: Jill has the utility function $U = lnI$, where I denotes the payoff. Which investment will she choose? (Note: Also show your response numerically).

Given that $ln(x)$ is a concave function, Jill will choose the less risky investment (i.e., the one with a lower standard deviation). Therefore she will choose investment A. One can also verify numerically that investment A gives her a higher utility level.

$$E(U(I_A)) = 0.2 \times ln(150) + 0.6 \times ln(200) + 0.2 \times ln(250) = 5.29 \quad (18)$$
$$E(U(I_A)) = 0.3 \times ln(150) + 0.4 \times ln(200) + 0.3 \times ln(250) = 5.28 \quad (19)$$

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