Calculus Handout - EC 308, Section 004
September 18, 2007

A. The derivative of a function at a particular point equals the slope of the function at that point. Notation: $f'(x) = \frac{\partial f(x)}{\partial x}$. Example: $f(x) = a + bx^2$. Then, the slope of the function $f(x)$ at a point, say x_0 will be: $f'(x_0) = \frac{\partial f(x)}{\partial x} |_{x=x_0} = 2bx_0$

B. Some differentiation rules

Consider two functions $f(x)$ and $g(x)$. Then we have the following rules:

- Multiplication: $(bf(x))' = bf'(x)$
- Sum: $(f(x) + g(x))' = f'(x) + g'(x)$
- Product: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
- Quotient: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- Chain rule: $[f(g(x))]' = f'(g(x))g'(x)$

C. Some differentiation formulas

1. $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
2. $f(x) = ln(x) \Rightarrow f'(x) = 1/x$
3. $f(x) = e^x \Rightarrow f'(x) = e^x$

D. Maxima and Minima

When a function f attains a maximum at the point x_0, then one needs to verify that:

$f'(x_0) = \frac{\partial f(x)}{\partial x} |_{x=x_0} = 0$ and $f''(x_0) = \frac{\partial f(x)}{\partial x^2} |_{x=x_0} < 0$,

where $f''(x)$ denotes the second derivative of the function. At a minimum, the last inequality is reversed.

E. Maxima under Constraints

To find x_0 and y_0 that maximize $f(x, y)$ under the condition that $y = g(x)$, one needs to insert g into f to get $h(x) = f(x, g(x))$ and then proceed as under **D** above.

Example: Let $f(x, y) = xy$, $g(x) = (c - bx)/a$, where $a > 0, b > 0, c > 0$. Then $h(x) = x(c - bx)/a$, $h'(x) = (c - 2bx)/a$, $h''(x) = -2b/a$, and $x_0 = \frac{c}{2b}$, $y_0 = \frac{c}{2a}$, respectively.