

HOMEWORK 4 - Solutions

Intermediate Microeconomics EC 308-004
November 7, 2007

Name: _____

by writing my name i swear by the honor code

Read all of the following information before starting the Assignment:

- You are allowed to work together on the homework. However, when it comes time for you to write up the solutions, you are required to do this on your own.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This assignment has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the answers!
- This assignment is due next Thursday, November 8 in class.
- Good luck!

1. (20 points) PROBLEM 1: This question has 6 parts. Suppose that a firm's cost function is $C(q) = 4q^2 + 16$.

a. (5 pts) PART A: Find variable cost, fixed cost, average cost, average variable cost, and average fixed cost.

$$VC = 4q^2 \quad (1)$$

$$FC = 16 \quad (2)$$

$$AC = \frac{C(q)}{q} = 4q + \frac{16}{q} \quad (3)$$

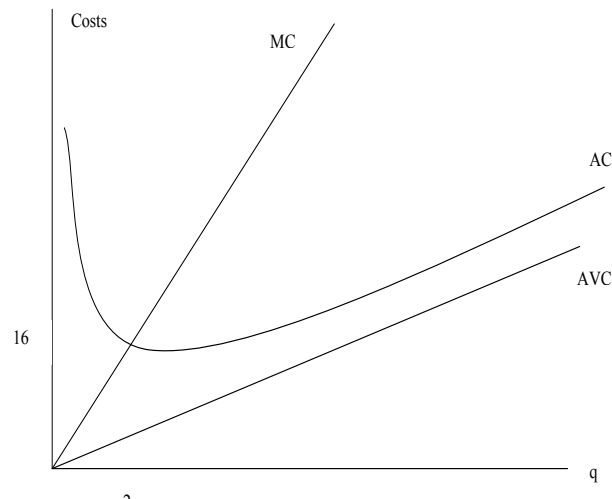
$$AVC = \frac{VC}{q} = 4q \quad (4)$$

$$AFC = \frac{FC}{q} = \frac{16}{q} \quad (5)$$

$$MC = 8q \quad (6)$$

b. (5 pts) PART B: Show the average cost, marginal cost, and average variable cost curves on a graph.

AC is u-shaped. AC is relatively large at first because the firm is not able to spread the fixed cost over many units of output. As output increases, AFC will fall relatively rapidly. AC will increase at some point because the AFC will become very small and AVC is increasing as q increases. AVC will increase because of diminishing returns to the variable factor labor. MC and AVC are linear, and both pass through the origin. AVC is everywhere below AC. MC is everywhere above AVC. If the average is rising, then the marginal must be above the average. MC will hit the AC at its minimum point.



c. (5 pts) PART C: Find the output that minimizes average cost.

The minimum AC quantity is where $MC = AC$:

$$AC = 4q + \frac{16}{q} = 8q = MC \Rightarrow \frac{16}{q} = 4q \Rightarrow 16 = 4q^2 \Rightarrow q = 2. \quad (7)$$

d. (5 pts) PART D: At what range of prices will the firm produce a positive output?

The firm will supply positive levels of output as long as $P = MC > AVC$, or as long as the firm is covering its variable costs of production. In this case, MC is everywhere above AVC so the firm will supply positive output at any positive price.

e. (5 pts) PART E: At what range of prices will the firm produce a positive output?

The firm will earn negative profit when $P = MC < AC$, or at any price below minimum AC. In part C above we found that the minimum AC quantity was $q = 2$. Plug $q = 2$ into the average cost function to find $AC = 16$. The firm will therefore earn negative profit if below 16.

f. (5 pts) PART F: At what range of prices will the firm produce a positive profit?

In part E we found that the firm would earn negative profit at any price below 16. The firm therefore earns positive profit as long as the price is above 16.

2. (20 points) PROBLEM 2: This problem has 3 parts.

a. (10 pts) PART A: Suppose that a firm's production function is $q = 9\sqrt{x}$ in the short run, where there are fixed costs of \$1000, and x is a variable input where cost is \$4,000 per unit. What is the total cost of producing a level of output q ? In other words, identify the total cost function $C(q)$.

The total cost function is:

$$C(x) = FC + VC = 1000 + 4000x. \quad (8)$$

Now rewrite the production function to express x in terms of q so that $x = \frac{q^2}{81}$. We can then substitute this into the above cost function to find $C(q)$:

$$C(q) = \frac{4000q^2}{81} + 1000. \quad (9)$$

b. (5 pts) PART B: Write down the equation for the supply curve.

The firm supplies output where $P = MC$ so the marginal cost curve is the supply curve, or $P = \frac{8000q}{81}$.

c. (5 pts) PART C: If price is \$1000, how many units will the firm produce? What is the level of profit? Illustrate on a cost curve graph.

To figure this out, set price equal to marginal cost to find:

$$P = \frac{8000q}{81} = 1000 \Rightarrow q = 10.125. \quad (10)$$

Profit is $\pi = 1000 \times 10.125 - (1000 + (4000 \times 10.125^2)/81) = 4062.5$. Graphically, the firm produces where the price line hits the MC curve. Since profit is positive, this will occur at a quantity where the price is greater than AC. To find profit on the graph, take the difference of the revenue box ($P \times q$) and the cost box ($AC \times q$).

3. (20 points) PROBLEM 3: The following problem has 4 parts. Suppose you are given the following information about a particular industry:

$$\text{Market demand : } Q^D = 6500 - 100P \quad (11)$$

$$\text{Market supply : } Q^S = 1200P \quad (12)$$

$$\text{Firm's total cost function : } C(q) = 722 + \frac{q^2}{200} \quad (13)$$

$$\text{Firm's marginal cost function : } MC(q) = \frac{2q}{200} \quad (14)$$

$$(15)$$

Assume all firms are identical and that the market is characterized by pure competition.

a. (5 pts) PART A: Find the equilibrium price, equilibrium quantity, the output supplied by the firm, and the profit of each firm.

Equilibrium price and quantity are found by setting market supply equal to market demand, so that $6500 - 100P = 1200P$. Solve to find $P = 5$ and substitute into either equation to find $Q = 6000$. To find the output for the firm set $P = MC$ so that $5 = \frac{2q}{200}$ and $q = 500$. Next:

$$\pi = Pq - C(q) = 5 \times 500 - 722 - \frac{500^2}{200} = 528. \quad (16)$$

Notice that since the total output in the market is 6000, and the firm output is 500, there must be $6000/500 = 12$ firms the industry.

b. (5 pts) PART B: Would you expect to see entry into or exit from the industry in the long-run? Explain. What effect will entry or exit have on market equilibrium?

Entry because the firms in the industry are making positive profit. As firms enter, the supply curve for the industry will shift down and to the right and the equilibrium price will fall, all else the same. This will reduce each firm's profit down to zero until there is no incentive for further entry.

c. (5 pts) PART C: What is the lowest price at which each firm would sell its output in the long run? Is profit positive, negative, or zero at this price? Explain.

In the long run the firm will not sell for a price that is below minimum AC. At any price below minimum AC, profit is negative and the firm is better off selling its fixed resources and producing zero output. To find the minimum average cost, set $MC = AC$ and solve for q :

$$\frac{2q}{200} = \frac{722}{q} + \frac{q}{200} \Rightarrow \frac{q}{200} = \frac{722}{2} \Rightarrow q^2 = 722 \times 200 \Rightarrow q = 380 \quad (17)$$

Also, $AC(380) = \frac{722}{380} + \frac{380}{200} = 3.8$.

d. (5 pts) PART D: What is the lowest price at which each firm would sell its output in the short run? Is profit positive, negative, or zero at this price? Explain.

The firm will sell for any positive price, because at any positive price MC will be above AVC (where $AVC = \frac{q}{2000}$). Profit is negative as long as $P < AC$, or as long as price is below 3.8.

$$AVC = \frac{q}{2000} < MC = \frac{2q}{2000} \quad (18)$$

4. (15 points) PROBLEM 4: Suppose the market for widgets can be described by the following equations:

$$\text{Demand : } P = 10 - Q \quad (19)$$

$$\text{Supply : } P = Q - 4 \quad (20)$$

a. (5 pts) PART A: What is the equilibrium price and quantity?

$$10 - Q = Q - 4 \Rightarrow Q^* = 7, \text{ and } P^* = 3. \quad (21)$$

b. (5 pts) PART B: Suppose the government imposes a tax of \$1 per unit to reduce widget consumption and raise government revenues. What will the new Q be? What price will the buyer pay? What amount will the seller receive?

With the imposition of a \$1.00 tax per unit, the demand curve shifts inward. At each price, the consumer wishes to buy less. Algebraically, the new demand function is:

$$P = 9 - Q \quad (22)$$

The new equilibrium is:

$$9 - Q = Q - 4 \Rightarrow Q^* = 6.5. \quad (23)$$

The buyer's price is $P_B^* = 10 - 6.5 = \$3.50$ and the seller's price is $P_S^* = 6.5 - 4 = \$2.50$.

c. (5 pts) PART C: Suppose the government has a change of heart about the importance of widgets to the happiness of the American public. The tax is removed and a subsidy of \$1.00 per unit granted to widget producers. What will the new Q be? What price will the buyer pay? What amount will the seller receive? What will the total cost to government be?

With a subsidy, the supply curve shifts outward. Remember that the supply curve for a firm is its MC. With a subsidy, the MC curve shifts down by the amount of the subsidy. The new supply is $P = Q - 5$. The new equilibrium will be:

$$10 - Q = Q - 5 \Rightarrow Q^* = 7.5. \quad (24)$$

The buyer pays \$2.50 and the seller receives \$3.50 (including the subsidy). With $Q = 7500$ and $s = \$1.00$, the total cost to government will be \$7,500.

5. (15 points) PROBLEM 5: Among the tax proposals regularly considered by Congress is an additional tax on distilled liquors. The tax would not apply to beer. The price elasticity of supply is 2.0, and the price elasticity of demand is -0.5. The cross price elasticity of demand for mixing drinks with respect to the price of liquor is -0.1.

a. (10 pts) PART A: If the new tax is imposed, who will bear the greater burden? Why?

Consumers will bear:

$$\frac{E_S}{E_S - E_D} = \frac{2}{2 - (-0.5)} = \frac{2}{2.5} = 0.8 \quad (25)$$

that is, 80% of the tax burden. This is true because supply is relatively elastic and demand is relatively inelastic.

Sellers support:

$$\frac{-E_B}{E_S - E_D} = \frac{-0.5}{2 - (-0.5)} = \frac{-0.5}{2.5} = 0.2 \quad (26)$$

which represents 20% of the tax burden.

b. (5 pts) PART B: Assuming that mixing drinks supply is infinitely elastic, how will the new tax affect the mixing drinks market?

Given that a 1% increase in the price of liquor leads to a 0.1% decrease in the demand for mixing drinks, the demand for the latter will shift to the left. However with a flat supply curve, the price of mixing drinks remains unchanged.

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