HOMEWORK 3 - Solutions

Intermediate Microeconomics EC 308-004
November 1, 2007

Name: ______________________
by writing my name i swear by the honor code

Read all of the following information before starting the Assignment:

- You are allowed to work together on the homework. However, when it comes time for you to write up the solutions, you are required to do this on your own.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This assignment has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the answers!
- This assignment is due next Thursday, October 25 in class.
- Good luck!

1. (*20 points*) PROBLEM 1: This question has 3 parts:

# of Workers	# of Chairs	Average Product (AP)	Marginal Product (MP)
1	10	10	10
2	18	9	8
3	24	8	6
4	28	7	4
5	30	6	2
6	28	4.7	-2
7	25	3.6	-3

a. (*10 pts*) PART A: Calculate the marginal and average product of labor for this production function.
For instance, when there are 2 workers, $AP = \frac{18}{2} = 9$ and $MP = \frac{18-10}{2-1} = 8$.

b. (*5 pts*) PART B: Does this production function exhibit diminishing returns to labor? Explain.
Yes. Returns to labor even become negative.

c. (*5 pts*) PART C: Explain intuitively what might cause the marginal product of labor to become negative.

Possibly, congestion in the chair manufacturer's factory could be at play.

2. (*20 points*) PROBLEM 2: Fill in the gaps in the table below.

Input Units	Total Output	Marginal Product (MP)	Average Product (AP)
0	0	-	-
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	-15	225

3. (*20 points*) PROBLEM 3: Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant? (Hint: For the second question take the first derivative of the MP with respect to that factor).

a. (*5 pts*) PART A: $q(K, L) = 3L + 2K$

There are at least two ways to answer these questions. First, following a more theoretical approach one can write:

$$q(tK, tL) = 3tK + 3tK = t(3K + 3L) = tq(K, L), t > 1 \tag{1}$$

which implies that the function displays constant returns to scale (CRS for short). Another way is to use numerical examples. For instance, whenever one doubles the inputs and the output doubles, then a function displays CRS. If the output more than doubles when you double the inputs, then the function displays increasing returns to scale (IRS for short). On the contrary, if the output less than doubles, then the function has decreasing returns to scale (DRS for short). Let's use $K = 2$ and $L = 2$. Then: $q(2, 2) = 10$. When $K = 4$ and $L = 4$, then $q(4, 4) = 20$. Since q doubles when we double K and L, then this function displays CRS.
Second question actually asks to verify whether the MP_K and MP_L are constant, increasing or decreasing. Mathematically, it requires us to take the first derivative of the MP with respect to the inputs (or difference the function q twice). First, we have:

$$MP_K = \frac{\partial q(K, L)}{\partial K} = 2 \tag{2}$$

and

$$MP_L = \frac{\partial q(K, L)}{\partial L} = 3. \tag{3}$$

Next, it is easy to see that the MPs are constant (i.e., $\frac{\partial MP_K}{\partial K} = 0$ and $\frac{\partial MP_L}{\partial L} = 0$).

b. (*5 pts*) PART B: $q(K, L) = (2L + 2K)^{1/2}$
For all $t > 1$ we have:

$$q(tK, tL) = \sqrt{2tL + 2tK} = \sqrt{t}\sqrt{2L + 2K} < t\sqrt{2L + 2K} = tq(K, L) \tag{4}$$

which implies that the function displays DRS. Alternatively, one can verify numerically that

$$q(4, 4) = \sqrt{16} = 4 < 2 \times q(2, 2) = 2\sqrt{8}. \tag{5}$$

Next:

$$MP_K = \frac{\partial q(K, L)}{\partial K} = \frac{1}{\sqrt{2L + 2K}} \tag{6}$$

$$MP_L = \frac{\partial q(K, L)}{\partial L} = \frac{1}{\sqrt{2L + 2K}} \tag{7}$$

and one can see that:

$$\frac{\partial MP_K}{\partial K} = \frac{\partial^2 q(K, L)}{\partial K^2} = -\frac{1}{\sqrt{(2L + 2K)^3}} < 0 \tag{8}$$

$$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 q(K, L)}{\partial L^2} = -\frac{1}{\sqrt{(2L + 2K)^3}} < 0 \tag{9}$$

Thus, both MPs are decreasing.

c. (*5 pts*) PART C: $q(K,L) = 3LK^2$
For all $t > 1$ we have:

$$q(tK,tL) = 3(tL)(tK)^2 = 3t^3LK^2 > t3LK^2 = tq(K,L) \tag{10}$$

In this case, the function displays IRS.
Next:

$$MP_K = \frac{\partial q(K,L)}{\partial K} = 6LK \tag{11}$$
$$MP_L = \frac{\partial q(K,L)}{\partial L} = 3K^2 \tag{12}$$

and one can see that:

$$\frac{\partial MP_K}{\partial K} = 6L > 0 \tag{13}$$
$$\frac{\partial MP_L}{\partial L} = 0 \tag{14}$$

Thus, MP_K is increasing while MP_K is constant.

d. (*5 pts*) PART D: $q(K,L) = L^{1/2}K^{1/2}$
For all $t > 1$ we have:

$$q(tK,tL) = \sqrt{tL}\sqrt{tK} = t\sqrt{K}\sqrt{L} = tq(K,L) \tag{15}$$

In this case, the function displays CRS.
Next:

$$MP_K = \frac{\partial q(K,L)}{\partial K} = \frac{\sqrt{L}}{2\sqrt{K}} \tag{16}$$
$$MP_L = \frac{\partial q(K,L)}{\partial L} = \frac{\sqrt{K}}{2\sqrt{L}} \tag{17}$$

and one can see that:

$$\frac{\partial MP_K}{\partial K} = -\frac{\sqrt{L}}{4\sqrt{K^3}} < 0 \tag{18}$$
$$\frac{\partial MP_L}{\partial L} = -\frac{\sqrt{K}}{4\sqrt{L^3}} < 0 \tag{19}$$

Thus, both MPs are decreasing.

4. (*20 points*) PROBLEM 4: The production function for a product is given by $q(K, L) = 100KL$. If the price of capital is \$120 per day and the price of labor \$30 per day, what is the minimum cost of producing 1000 units of output? (Hint: Set-up the Lagrangian to minimize $C = wL + rK$ subject to the constraint that $q(K, L) = 1000$ and solve for K and L in terms of C_0).
Objective: minimize $C = wL + rK$ such that $q(K, L) = 1000$.

Step 1. Write the Lagrangian:

$$L = 30L + 120K - \lambda[100KL - 1000] \tag{20}$$

Step 2. Take the FOCs:

$$\frac{\partial L}{\partial L} = 30 - \lambda 100K = 0 \rightarrow \lambda = \frac{30}{100K} \tag{21}$$

$$\frac{\partial L}{\partial K} = 120 - \lambda 100L = 0 \rightarrow \lambda = \frac{120}{100L} \tag{22}$$

$$\frac{\partial L}{\partial \lambda} = 100KL - 1000 = 0 \tag{23}$$

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Step 3. From the first two equations find that $L = 4K$. Plug this in the last equation to find $100 \times 4K^2 = 1000$ which solves for $K = 1.58$, and $L = 6.32$. Overall, the minimum cost that one obtains is $C = 30 \times 6.32 + 120 \times 1.58 = \$379.20$.

5. (*20 points*) PROBLEM 5: Suppose a production function is given by $F(K, L) = KL^2$; the price of capital is \$10 and the price of labor is \$15. What combination of labor and capital maximizes the output given a fixed cost of C_0? (Hint: Set-up the Lagrangian to maximize $q(K, L)$ subject to the constraint that $wL + rK = C_0$).

Objective: maximize $q(K, L)$ subject to $wL + rK = C_0$.

Step 1. Write the Lagrangian:

$$L = KL^2 - \lambda[15L + 10K - C_0] \tag{25}$$

Step 2. Take the FOCs:

$$\frac{\partial L}{\partial L} = L^2 - 10\lambda = 0 \rightarrow \lambda = \frac{L^2}{10} \tag{26}$$

$$\frac{\partial L}{\partial K} = 2KL - 15\lambda = 0 \rightarrow \lambda = \frac{2KL}{15} \tag{27}$$

$$\frac{\partial L}{\partial \lambda} = 15L + 10K - C_0 = 0 \tag{28}$$

$$\tag{29}$$

Step 3. From the first two equations find that $L = \frac{4K}{3}$. Plug this in the last equation to find that:

$$15 \times \frac{4K}{3} + 10K - C_0 = 0 \rightarrow 30K = C_0 \tag{30}$$

Thus $K = \frac{C_0}{30}$ and $L = \frac{2C_0}{45}$.

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