Appears in G. Dionne, *Handbook of Insurance*. Kluwer (2000)

May 1999

The Theory of Insurance Demand

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The theory of insurance demand is often regarded as the purest example of economic behavior under uncertainty. Interestingly, whereas a decade ago most upper-level textbooks on microeconomics barely touched on the topic of uncertainty, much less insurance demand, textbooks today often devote substantial space to the topic. The purpose of this chapter is to present the basic model of insurance demand, that imbeds itself not only into the other papers in this volume and in the insurance literature, but also in many other settings within the finance and economics literatures. Since models that deal with nonexpected utility analysis are dealt with elsewhere in this volume, I focus only on the expected-utility framework.

If we were to view insurance as simply a case of optimal risk sharing, we would be led to a simple sharing rule due to Karl Borch (1962). However, for many reasons, not the least of which is the sheer size of the economy, such ideal risk sharing rarely seems to take place. Indeed, even Borch himself had to move from the level of the individual, past the level of the insurance company, and to the level of reinsurance in expositing his classic result. In this sense, we can view insurance as an intermediary. Although contingent contracts that allow for mutual risk sharing would be first best, such contracts are not feasible. We thus see insurers in the economy, who approximate the process by gathering and pooling the risks of a large number of individuals.

The device offered by the insurer is one in which, for a fixed premium, the insurer offers an indemnity for incurred losses. Of course, there are many variations on this theme, as one can see from gleaning the pages of this volume. From a purely theoretical viewpoint, the model presented in section 1 of this chapter should be viewed as a base model, from which all other models deviate.

In some ways, insurance is simply a financial asset. However, whereas most financial assets are readily tradable and have a risk that relates to the marketplace, insurance is a contract contingent on an individual's own personal wealth changes. This personal nature of insurance is what distinguishes it from other financial assets. It also exacerbates problems of informational asymmetry, such as moral hazard and adverse selection, which also are dealt with elsewhere in this volume.

The preponderance of insurance models isolate the insurance-purchasing decision. The consumer decides how much insurance to buy for a well-defined risk. And indeed, this chapter starts out the same was in section 1. However, when multiple risks face the consumer, it is not likely to be optimal to decide how to handle each risk separately. Rather, some type of overall risk-management strategy is called for. Even if we make an insurance decision in isolation, the presence of these other risks is most likely going to affect our choice. The second part of this chapter shows how the presence of other risks – so-called "background risk" – impacts the consumer's insurance-purchasing decision.

1. The Singe Risk Model

Insurance contracts themselves can be quite complicated, but the basic idea is fairly simple. For a fixed premium P the insurer will pay the insured a contingent amount of money, that depends upon the value of a well-defined loss. This insurance payment is referred to as the *indemnity*.

To make the model concrete, consider an individual with initial wealth $W > 0$. Let the random variable $\tilde{x}$ denote the amount of the loss, where $0 \leq x \leq W$. The insurance indemnity is contingent only on x and will be written as $I(x)$. We often assume that $I(x)$ is nondecreasing in x and that $0 \leq I(x) \leq x$, though neither of these assumptions is necessary to develop a theory of insurance demand. We do, however, assume that the realization of $\tilde{x}$ is costlessly observable by all parties and that both parties agree on the distribution of the random variable $\tilde{x}$. Models that do not make these last two assumptions are dealt with elsewhere in this volume.

The insurer, for our purpose, can be considered as a risk-neutral firm that charges a market-determined price for its product. The individual is considered to be risk averse with von Neumann-Morgenstern utility of final wealth given by the function $u(\cdot)$, where u is assumed to be everywhere twice differentiable with $u' > 0$ and $u'' < 0$. The assumption of differentiability is not innocuous. It is tantamount in our model to assuming that risk aversion is everywhere of order 2.[1]

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[1] See Segal and Spivak (1990). Although extensions to the case where u is not everywhere differentiable are not difficult, they are not examined here. See Schlesinger (1997) for some basic results.

1.1 Proportional Coinsurance

The simplest type of indemnity payment is one in which the insurer pays a fixed proportion, say α, of the loss. Thus, $I(x) = \alpha x$. This type of insurance indemnity is often referred to as *coinsurance*, since the individual retains (or "coinsures") a fraction $1-\alpha$ of the loss. If $\alpha = 1$, the insurer pays an indemnity equal to the full value of the loss and the individual is said to have *full insurance.*

An assumption that $0 \leq I(x) \leq x$ here is equivalent to assuming that $0 \leq \alpha \leq 1$. The case where $\alpha > 1$ is often referred to as *over insurance*. The case where $\alpha < 0$ is referred to by some as "selling insurance," but this description is incorrect. If $\alpha < 0$, the individual is taking a short position in his or her *own* loss; whereas selling insurance is taking a short position in someone else's loss.

To consider the insurance-purchasing decision, we need to specify the insurance premium as a function of the indemnity. The most general form of the premium is

$$(1) \qquad \mathrm{P}[I(\cdot)] = E[I(\tilde{x}) + c[I(\tilde{x})]].$$

Here E denotes the expectation operator and $c(\cdot)$ is a cost function, where $c[I(x)]$ denotes the cost of paying indemnity $I(x)$, including any market-based charges for assuming the risk $I(\tilde{x})$. Note that P itself is a *functional*, since it depends upon the function $I(\cdot)$.

As a base case, we often consider $c[I(x)] = 0 \ \forall x$. This case is usually referred to as the case of *perfect competition* in the insurance market, since it implies that insurers receive an expected profit of zero, and the premium is referred to as a *fair premium*.[2]

The premium, as defined in (1), is a bit too general to suit our purpose here. See Gollier (1999) for more discussion of this general premium form. We consider here the simplest case of (1) in which the expected cost is proportional to the expected indemnity; in particular

$$(2) \qquad \mathrm{P}(\alpha) = E(\alpha\tilde{x} + \lambda\alpha\tilde{x}) = \alpha(1+\lambda)E\tilde{x}\ ,$$

where λ is called the *loading factor*, $\lambda \geq 0$. The individual's final wealth can then be expressed as a random variable, dependent upon the choice of α,

$$(3) \qquad \tilde{Y}(\alpha) \equiv W - \alpha(1+\lambda)E\tilde{x} - \tilde{x} + \alpha\tilde{x}\ .$$

The individual's objective is choose α so as to maximize his or her expected utility

$$(4) \qquad \underset{\alpha}{\max\text{imize}}\ \mathrm{E}[\mathrm{u}(\tilde{\mathrm{Y}}(\alpha))],$$

where we might or might not wish to impose the constraint that $0 \leq \alpha \leq 1$.

Solving (4) is relatively straightforward, yielding a first-order condition for the unconstrained objective

$$(5) \qquad \frac{dEu}{d\alpha} = E[u'(\tilde{Y}(\alpha)) \cdot (\tilde{x} - (1+\lambda)E\tilde{x})]\ .$$

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[2] Obviously real-world costs include more than just the indemnity itself, plus even competitive insurers earn a "normal return" on their risk. Thus, we do not really expect $c[I(x)] = 0$. However, real-world markets also allow for the insurer to invest premium income, which is omitted here, so that zero-costs might not be a bad approximation for our purpose of developing a simple model. The terminology "fair premium" is taken from the uncertainty literature, since such a premium in return for the random payoff $I(\tilde{x})$ represents a "fair bet" for the insurer.

The second-order condition for a maximum holds trivially from our assumption that $u'' < 0$. Indeed, $d^2Eu/d\alpha^2$ is negative everywhere, indicating that any α^* satisfying (5) will be a global maximum. The fact that $E[u(\widetilde{Y}(\alpha))]$ is globally concave in α also turns out to be key in later examining various comparative statics.

Evaluating $dEu/d\alpha$ at $\alpha = 1$ shows that

$$\text{(6)} \qquad \left.\frac{dEu}{da}\right|_{\alpha=1} = -\lambda Eu'(\widetilde{Y}(1)) \cdot E\widetilde{x} + Cov(u'(\widetilde{Y}(1)), \widetilde{x}) = -\lambda Eu'(\widetilde{Y}(1)) \cdot E\widetilde{x} + 0,$$

where $Cov(\cdot,\cdot)$ denotes the covariance operator. Consequently, the sign of (6) will be zero if $\lambda = 0$ and will be negative if $\lambda > 0$. Together with the concavity of $Eu(\widetilde{Y}(\alpha))$ in α, this implies the following result, usually referred to as *Mossin's Theorem*:[3]

Theorem: *If proportional insurance is available at a fair price* $(\lambda = 0)$*, then full coverage* $(\alpha^* = 1)$ *is optimal. If the price of insurance includes a positive premium loading* $(\lambda > 0)$*, then partial insurance* $(\alpha^* < 1)$ *is optimal.*

Note that Mossin's Theorem does not preclude a possibility that $\alpha^* \leq 0$ in the unconstrained case. Indeed, evaluating $dEu/d\alpha$ at $\alpha = 0$ when $\lambda > 0$, yields

$$\text{(7)} \qquad \frac{dEu}{d\alpha}\Big|_{\alpha=0} = -\lambda Eu'(\widetilde{Y}(0)) \cdot E\widetilde{x} + Cov(u'(\widetilde{Y}(0), \widetilde{x}).$$

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[3] The result is often attributed to Mossin (1968), with a similar analysis also appearing in Smith (1968).

Since the covariance term in (7) is positive and does not depend on λ, we note that there will exist a unique value of λ such that the derivative in (7) equals zero. At this value of λ, zero coverage is optimal, $\alpha^* = 0$. For higher values of λ, $\alpha^* < 0$. Since $Eu(\tilde{Y}(\alpha))$ is concave in α, $\alpha = 0$ will be a constrained optimum whenever the unconstrained optimum is negative. In other words, if the price of insurance is too high, the individual will not purchase any insurance.

As long as the premium loading is nonnegative, $\lambda \geq 0$, the optimal level of insurance will be no more than full coverage, $\alpha^* \leq 1$. If, however, we allow for a negative premium loading, $\lambda < 0$, such as might be the case when the government subsidizes a particular insurance market, then over insurance, $\alpha^* > 1$, will indeed be optimal in the case where α is unconstrained. Strict concavity of $\mathrm{Eu}(\tilde{\mathrm{Y}}(\alpha))$ in α once again implies that full insurance, $\alpha = 1$, will be a constrained optimum for this case, when over insurance is not allowed.

It may be instructive for some readers to compare the above results with the so-called *portfolio problem* in financial economics. The standard portfolio problem has an investor allocate her wealth between a risky and a riskless asset. If we let A denote final wealth when all funds are invested in a riskless asset, and let $\tilde{\mathrm{z}}$ denote the random excess payoff above the payoff on the riskless asset, the individual must choose a weight β, such that final wealth is

$$(8) \qquad \mathrm{Y}(\beta) = (1-\beta)\mathrm{A} + \beta(\mathrm{A} + \tilde{\mathrm{z}}) = \mathrm{A} + \beta\ \tilde{\mathrm{z}}\ .$$

A basic result in the portfolio problem is that $\mathrm{sgn}\ \beta^* = \mathrm{sgn}\ \mathrm{E}\tilde{\mathrm{z}}$. If we set $\mathrm{A} \equiv \mathrm{W} - (1+\lambda)\mathrm{E}\tilde{\mathrm{x}}$, $\tilde{\mathrm{z}} \equiv (1+\lambda)\mathrm{E}\tilde{\mathrm{x}} - \tilde{\mathrm{x}}$, and $\beta = (1-\alpha)$, then (8) is equivalent to (3).

Noting that $\mathrm{sgn}\ \mathrm{E}\tilde{z} = \mathrm{sgn}\ \lambda$ in this setting, our basic portfolio result is exactly equivalent to Mossin's Theorem. Using equation (8), we can think of the individual starting from a position of full insurance $(\beta = 0)$ and then deciding upon the optimal level to coinsure, β^*. If $\lambda > 0$, then coinsurance has a positive expected return, so that any risk averter would choose $\beta^* > 0$ (i.e. $\alpha^* < 1$).

1.2 Effects of Changes in Wealth and Price

In the general case, it is often difficult to define what is meant by the *price* and the *quantity* of insurance. Since the indemnity is a function of a random variable and since the premium is a functional of this indemnity function, both price and quantity – the two fundamental building blocks of economic theory – have no direct counterparts for insurance. However, for the case of coinsurance, we have the level of coinsurance α and the premium loading factor λ, which fill in nicely as proxy measures of quantity and price respectively.

If the individual's initial wealth changes, but the loss exposure remains the same, will more or less insurance be purchased? In other words, is insurance a "normal" or an "inferior" good? Clearly, if $\lambda = 0$, then Mossin's Theorem implies that full insurance remains optimal. So let us consider the case where $\lambda > 0$, but assume that λ is not too large, so that $0 < \alpha^* < 1$. Since $\mathrm{Eu}(\tilde{Y}(\alpha))$ is concave in α, we can determine the effect of a higher W by differentiating the first-order condition (5) with respect to W. Before doing this however, let us recall a few items from the theory of risk aversion.

If the Arrow-Pratt measure of local risk aversion, $\mathrm{r(y)=-u''(y)/u'(y)}$, is decreasing in wealth level y, then preferences are said to exhibit decreasing absolute risk aversion (DARA). Similarly, we can define constant absolute risk aversion (CARA) and increasing absolute risk aversion (IARA). We are now ready to state the following result.

Proposition 1: *Let the insurance loading λ be positive. Then for an increase in the initial wealth level W,*

- *(i) the optimal insurance level α^* will decrease under DARA,*
- *(ii) the optimal insurance level α^* will be invariant under CARA,*
- *(iii) the optimal insurance level α^* will increase under IARA.*

Proof: Let F denote the distribution of $\tilde{\mathrm{x}}$. By assumption, the support of F lies in the interval [0,W]. Define $\mathrm{x_0 \equiv (1+\lambda)E\tilde{x}}$. Assume DARA. Then we note that $\mathrm{r(y_1) < r(y_0) < r(y_2)}$ for any $\mathrm{y_1 > y_0 > y_2}$, and, in particular for $\mathrm{y_0 = W - \alpha^*(1+\lambda)E\tilde{x} - x_0 + \alpha x_0}$. Now

$$(9) \qquad \left.\frac{\partial^2 Eu}{\partial\alpha\partial W}\right|_{\alpha^*} = \int_0^W u''(Y(\alpha^*))(x-(1+\lambda)E\tilde{x})dF$$

$$= -\int_0^{\mathrm{x_0}} \mathrm{r(Y(\alpha^*))\,u'(Y(\alpha^*))(\,x-(1+\lambda)E\tilde{x})dF} -$$

$$\int_{x_0}^{W} r(Y(\alpha^*))u'(Y(\alpha^*))(x-(1+\lambda)E\tilde{x})dF$$

$$< -\mathrm{r(y_0)}\left[\int_0^{\mathrm{W}} \mathrm{u'(Y(\alpha^*))(x-(1+\lambda)E\tilde{x})dF}\right] = 0.$$

Thus increasing wealth causes $\alpha*$ to fall.

The cases where preferences exhibit CARA or IARA can be proved in a similar manner. ■

We should caution the reader that DARA, CARA and IARA do not partition the set of risk-averse preferences. Indeed each of these conditions is shown to be sufficient for the comparative-static effects in Proposition, though none is necessary.

The case of CARA is often used as a base case, since such preferences eliminate any income effect. However, a more common and, by most standards, realistic assumption is DARA, which implies that insurance is an inferior good. One must use caution in using this interpretation however. It is valid only for the case of a fixed loss exposure $\tilde{x}$. Since real-world loss exposures typically increase as wealth increases, we do not necessarily expect to see richer individuals spending less on their insurance purchases, *ceteris paribus*.[4] We do, however, expect that they would spend less on the same loss exposure.

In a similar manner, we can examine the effect of an increase in the loading factor λ on the optimal level of insurance coverage. Differentiating the first-order condition with respect to λ obtains

$$(10) \qquad \left.\frac{\partial^2 \mathrm{Eu}}{\partial\alpha\partial\lambda}\right|_{\alpha*} = -[\mathrm{E}\tilde{x}\mathrm{Eu}'(\tilde{Y}(\alpha*))] - \alpha\,\mathrm{E}\tilde{x}\frac{\partial^2 \mathrm{Eu}}{\partial\alpha\partial \mathrm{W}} .$$

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[4] If the support of $\tilde{x}$ is [0, L], it may be useful to define $W \equiv W_0 + L$. If the loss exposure is unchanged, an increase in W can be viewed as an increase in W_0. More realistically, an increase in W will consist of increases in both W_0 and L.

The first term on the right-hand side of equation (10) captures the substitution effect of an increase in λ. This effect is negative due to the higher price of insurance. The second term on the right-hand side of (10) captures an income effect, since a higher premium would lower overall wealth, *ceteris paribus*. For a positive level of α, which we are assuming, this effect will be the opposite sign of $\partial^2 \mathrm{Eu} / \partial\alpha\partial \mathrm{W}$. For example, under DARA, this income effect is positive: the price increase lowers the average wealth of the individual, rendering him or her more risk averse. This higher level of risk aversion, as we shall soon see, implies that the individual will purchase more insurance. If this second (positive) effect outweighs the negative substitution effect, insurance can be considered a Giffen good.[5] More comprehensively, the following result is a direct consequence of equation (10) and Proposition 1.

Proposition 2: *Let the insurance loading be positive, with* $0 < \alpha^* < 1$. *Then, insurance cannot be a Giffen good if preferences exhibit CARA or IARA, but may be Giffen if preferences exhibit DARA.*

1.3 Changes in Risk and in Risk Aversion

If the loss distribution F changes, it is sometimes possible to predict the change in optimal insurance coverage α^*. Conditions on changes to F that are both necessary and sufficient for α^* to increase are not trivial, but can be found

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[5] A necessary and sufficient condition for insurance not to be Giffen is given by Briys, Dionne and Eeckhoudt (1989).

by applying a Theorem of Gollier (1995) to the portfolio problem, and then using the equivalence of the portfolio problem and the insurance problem. Although this condition is very complex, there are several sufficient conditions for α^* to rise due to a change in risk that are relatively straightforward. Since this topic is dealt with elsewhere in this volume (Eeckhoudt and Gollier, 1999), I do not detour to discuss it any further here.

A change in risk aversion, on the other hand, has a well-defined effect upon the choice of insurance coverage. First of all, we note that for an insurance premium that is fair, $\lambda = 0$, any risk-averse individual will choose an insurance policy with full coverage, $\alpha^* = 1$. If, however, insurance premia include a positive premium loading, $\lambda > 0$, then an increase in risk aversion will always increase the level of insurance. More formally,

Proposition 3: *Let the insurance loading be positive, with $0 < \alpha^* < 1$. An increase in the individual's degree of risk aversion at all levels of wealth will lead to an increase in the optimal level of coverage, ceteris paribus.*

Proof: Let α^*_{u} denote the optimal level of coverage under the original utility function u. Let v denote a uniformly more risk-averse utility function. We know from Pratt (1964), that there exists a function $\mathrm{g}:[\mathrm{Image\ u}] \rightarrow \Re$ such that $\mathrm{v(y)} = \mathrm{g[u(y)]}$, where $\mathrm{g}' > 0$ and $\mathrm{g}'' < 0$.

Since v is a risk-averse utility function, we note that $Ev(\widetilde{Y}(\alpha))$ is concave in α. Thus, consider the following:

$$(11)\quad \left.\frac{dEv}{d\alpha}\right|_{\alpha_u^*} = \left.\frac{dEg[u]}{d\alpha}\right|_{\alpha_u^*} = \int_0^W g'[u(Y(\alpha_u^*))]u'(Y(\alpha_u^*))(x-(1+\lambda)E\widetilde{x})dF$$

$$> g'[u(y_0)]\left\{\int_0^{x_0} u'(Y(\alpha_u^*))(x-(1+\lambda)E\widetilde{x})dF + \int_{x_0}^W u'(Y(\alpha_u^*))(x-(1+\lambda)E\widetilde{x})dF\right\} = 0$$

where x_0 and y_0 are as defined in the proof of Proposition 1, and where the inequality follows from the concavity of g. This last expression equals zero by the first-order condition for α_u^*.

Since $Ev(\widetilde{Y}(\alpha))$ is concave in α, the inequality in (11) implies that $\alpha_v^* > \alpha_u^*$.

1.4 Self-Insurance and Self-Protection

It is useful, at this point, to distinguish insurance from two other types of protection against loss. These alternatives were first examined in a classic article by Ehrlich and Becker (1979) and represent engineering-types of alternatives. That is, while insurance, which Ehrlich and Becker distinguish under the label "market insurance," offers third-party indemnification for losses that occur, these alternatives actually change the frequency and/or severity of the loss distribution. In particular, self-insurance lowers the financial severity of any loss that occurs, whereas self-protection reduces the likelihood that a loss occurs.[6] An example of self-insurance might be the installation a sprinkler

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[6] This terminology is still standard in the economics literature. These two activities are typically referred to as "loss reduction" and "loss prevention" respectively in the insurance literature.

system to protect against fire damages. An example of self-protection might be the installation of dead-bolt locks at home to keep potential thieves from entering.

In reality, the distinction between self-insurance and self-protection is often blurred. Indeed even in the above examples, the sprinkler might extinguish a fire in a waste basket, essentially lowering the chance of any loss occurring. Likewise, the dead-bolt lock might only take away from some of the thief's time spent in your house, thus lowering the level of damages. The point is that most investment to control losses simultaneously contains some degree of both self-insurance and self-protection. Moreover, changes in a loss distribution are not typically decomposable into self-insurance and self-protection types of changes.[7]

One way to view self-insurance in the general case is to redefine the "indemnity function" $I(x)$ as the deterministic reduction of the loss, which would have been of size x without self-insurance.[8] Thus, a loss that would have been x is now reduced to the amount $x-I(x)$. Instead of a "premium" $P[I(\cdot)]$, we can view P as the cost for achieving the loss-reduction schedule $I(\cdot)$. In this setting, it is not surprising that self-insurance and market insurance are substitutes, which was proven formally by Ehrlich and Becker for the simple case where there are

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[7] Ehrlich and Becker (1979) perform only a sketchy analysis of continuous loss distributions, and they provide no clear definitions of self-insurance and self-protection except for the simple two-state framework.

[8] If the reduction in loss size is stochastic, rather than deterministic, the analysis becomes much more complex.

only two states of nature: loss and no-loss, where the loss size without self-insurance is fixed.

Similarly, one way to view self-protection is to define the random variable $\tilde{L}$ as the size of a loss, conditional on the occurrence of a loss. We then let p denote the probability of a loss occurring. The loss amount $\tilde{x}$ thus has a distribution that contains an atom at zero. In particular, no loss occurs with probability $(1-p)$, and with probability p the consumer experiences a loss of random size $\tilde{L}$. To introduce self-protection, let c denote the level of investment in this type of activity. We assume that the loss probability is affected with $p \equiv p(c)$, where $p(\cdot)$ is a decreasing function. Final wealth can thus be viewed as a compound lottery. With probability $1-p(c)$ the level of wealth is $W-c$, and with probability $p(c)$ final wealth is $W - c - \tilde{L}$, where the distribution of loss severity $\tilde{L}$ is assumed to be unaffected by c.

Whereas an investment in market insurance or in self-insurance will increase wealth in the “bad” states of nature at a cost of reduced wealth in the good states, the same cannot be said of self-protection. By increasing the expenditure on self-protection c, final wealth is lower in every state of nature. However, self-protection alters the probabilities so that the best state of nature (no loss) is more likely. Given their very different structures, it is not surprising that self-protection is not generally a substitute for market insurance or self-insurance, and may indeed be a complement.[9]

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[9] Ehrlich and Becker (1979) derive complementarily in a model with two states of nature, under certain cost conditions.

If we turn our attention to the effects of risk aversion on the purchase of self-insurance, it is not surprising that self-insurance behaves much like market insurance. Under reasonable cost conditions, investment in self-insurance increases with higher levels of risk aversion. The same is not true for self-protection. Indeed, since self-protection lowers wealth in *all* states of nature, including the state with the highest loss, a more risk averse individual might optimally invest *less* in self-protection, in order to improve the worst possible wealth level.[10]

1.5 Deductible Insurance

Although proportional coinsurance is the simplest case of insurance demand to model, real-world insurance contracts often include fixed co-payments per loss or deductibles. Indeed, optimal contracts include deductibles under fairly broad assumptions, and under fairly simple but realistic pricing assumptions, straight deductible policies can be shown to be optimal.[11] In this section, we examine a few aspects of insurance demand when insurance is of the deductible type.

For deductible insurance, the indemnity is set equal to the excess of the loss over some predetermined level. Let L denote the supremum of the support of the loss distribution, so that L denotes the maximum possible loss. By

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[10] Dionne and Eeckhoudt (1985) show these risk-aversion effects for the two-state model. Briys and Schlesinger (1990) extend this analysis by analyzing the effects of self-insurance and self-protection on the riskiness of final wealth. Sweeney and Beard (1992), show that there do not exist any conditions in an expected-utility framework that would lead to an individual always investing more in self-protection.

[11] See the essay by Gollier (1999) in this volume for a detailed analysis of the optimality of deductibles.

assumption, we have $L \leq W$. Define the deductible level $D \in [0, L]$ such that $I(x) \equiv \max(0, x - D)$. If $D=0$, the individual once again has full coverage, whereas $D=L$ now represents zero coverage. One complication that arises, is that the general premium, as given by equation (1), can no longer be written as a function of only the mean of the loss distribution, as in (2). Also, it is difficult to find a standard proxy for the *quantity* of insurance in the case of deductibles.[12]

In order to keep the model from becoming overly complex, we assume here that the distribution F is continuous, with density function f, so that $dF(x) = f(x)dx$. We will once again assume that the insurance costs are proportional to the expected indemnity, so that the premium for deductible level D is given by

$$(12) \qquad P(D) = (1+\lambda)E[I(\tilde{x})] = (1+\lambda)\int_D^L (x - D)dF(x)$$
$$= (1+\lambda)\int_D^L [1 - F(x)]dx\,,$$

where the last equality is obtained via integration by parts.

Using Leibniz Rule, one can calculate the marginal premium reduction for increasing the deductible level,[13]

$$(13) \qquad P'(D) = -(1+\lambda)(1 - F(D))\,.$$

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[12] Meyer and Ormiston (1998) make a strong case for using $E[I(\tilde{x})]$, although its often much simpler to use D as an inverse proxy for insurance demand.

[13] Leibniz rule states that

$$\frac{d}{dt}\int_{a(t)}^{b(t)} H(x,t)dx = H(b,t)b'(t) - H(a,t)a'(t) + \int_{a(t)}^{b(t)} \frac{\partial H}{\partial t}dx\,.$$

By increasing the deductible level, say by an amount ΔD, the individual receives a lower payout in all states of the world for which the loss exceeds the deductible. The likelihood of these states is $1 - F(D)$. While it is true that the likelihood will also change as D changes, this effect is of secondary importance and, due to our assumption of a continuous loss distribution, disappears in the limit.

Following the choice of a deductible level D and using the premium as specified in (12), final wealth can be written as

(14) $\tilde{Y}(D) = W - P(D) - \min(\tilde{x}, D)$.

The individual's objective is now to choose the best deductible,

(15) $\underset{D}{\max\text{imize}}\ E[u(\tilde{Y}(D))]$, where $0 \le D \le L$. Assume that the premium loading is nonnegative, $\lambda \ge 0$, but not so large that we obtain zero coverage as a corner solution, $D^* = L$. The first-order condition for the maximization in (15), again using Leibniz rule, is

$$(16)\quad \frac{dEu}{dD} = -P'\int_0^D u'(W - P - x)dF + (-P' - 1)\int_D^L u'(W - P - D)dF$$

$$= -P'\int_0^D u'(W - P - x)dF + (-P' - 1)(1 - F(D))u'(W - P - D) = 0.$$

The first term in either of the center expressions in (16) represents the marginal net utility benefit of premium savings from increasing D, conditional on the loss not exceeding the deductible level. The second term is minus the net marginal utility cost of a higher deductible, given that the loss exceeds the deductible. Thus, (16) has a standard economic interpretation of choosing D* such that marginal benefit equals marginal cost.

The second-order condition for the maximization in (16) can be shown to hold as follows.

$$(17)\qquad \frac{d^2Eu}{dD^2} = (1+\lambda)(-f(D))\int_0^D u'(W-P-x)dF + (-P')u'(W-P-D)f(D)$$

$$+(-P')^2\int_0^D u''(W-P-x)dF + (1+\lambda)(-f(D))(1-F(D))u'(W-P-D)$$

$$+(-P'-1)(-f(D))u'(W-P-D) + (-P'-1)^2(1-F(D))''(W-P-D).$$

Multiplying all terms containing $f(D)$ in (17) above by $(1-F(D))/(1-F(D))$ and simplifying, yields

$$(18)\qquad \frac{d^2Eu}{dD^2} = \frac{-f(D)}{1-F(D)}\left[-P'\int_0^D u'(W-P-x)dF + (-P'-1)(1-F(D))u'W-P-D)\right]$$

$$+\left[(-P')^2\int_0^D u''(W-P-x)dF + (-P'-1)^2(1-F(D))u''(W-P-D)\right] < 0$$

The first term in (18) is zero by the first-order condition, while the second term is negative from the concavity of u, thus yielding the inequality as stated in (18).

To see that Mossin's Theorem can be extended to the case of deductibles, rewrite the derivative in (16) as

$$(19)\qquad \frac{dEu}{dD} = (1-F(D))\left[(1+\lambda)\int_0^L u'(W-P-\min(x,D))dF - u'(W-P-D)\right].$$

If $\lambda = 0$, then (19) will be negative for any $D > 0$, and is easily seen to equal zero when $D = 0$. For $\lambda > 0$, (19) will be positive at $D = 0$, so that the deductible should be increased. Therefore, Mossin's Theorem also holds for a choice of deductible.

It also is straightforward to extend the comparative-static results of Propositions 1-3 to the case of deductibles as well, although we do not provide the details here.

2. The Model with Multiple Risks

Although much is to be learned from the basic single-risk model, rarely is the insurance decision made with no other uncertainty in the background. This so-called background risk might be exogenous or endogenous. In the latter case decisions on how to best handle risk cannot usually be decided in isolation on a risk-by-risk basis. Rather, some type of comprehensive risk management policy must be applied.[14] However, even in the case where the background risk is exogenous and independent of the insurable risk, we will see that the mere presence of background risk affects the individual's insurance choice.

The existence of uninsurable background risk is often considered a consequence of incomplete markets for risk sharing. For example, some types of catastrophic risk might contain too substantial an element of nondiversifiable risk, including a risk of incorrectly estimating the parameters of the loss

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[14] This question was first addressed by Mayers and Smith (1983) and Doherty and Schlesinger (1983). The special case of default risk was developed by Doherty and Schlesinger (1990).

distribution, to be insurable. Likewise, nonmarketable assets, such as one's own human capital, might not find ready markets for sharing the risk. Similarly, problems with asymmetry of information between the insurer and the insured, such as moral hazard and/or adverse selection, might preclude the existence of insurance markets for certain risks.

We begin the next section by examining a type of secondary risk that is always present for an insurable risk, but almost universally ignored in insurance theory; namely the risk that the insurer does not pay the promised indemnity following a covered loss. The most obvious reason for nonpayment is that the insurer may be insolvent and not financially capable of paying its claims in full. However, other scenarios are possible. For instance, there might be some events that void insurance coverage, such as a probationary period for certain perils to be included, or exclusion of coverage in situations of civil unrest or war.[15] Even if the insurer pays the loss in full, it may decide to randomly investigate a claim thereby substantially delaying payment. In such an instance, the delay reduces the present value of the indemnity, which has the same effect as paying something less than the promised indemnity.

2.1 The Model with Default Risk

We consider here an insurance model in which the insurer might not pay its claims in full. To keep the model simple, we consider only the case of a full

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[15] Although not modeled in this manner, the possibility of a probationary period is examined by Eeckhoudt, et al. (1988), who endogenize the length of probation.

default on an insured's claim in which a loss of a fixed size either occurs or does not occur. Let the support of the loss distribution be $\{0, \mathrm{L}\}$, where a loss of size L occurs with probability $\mathrm{p}, 0 < \mathrm{p} < 1$. Let α once again denote the share of the loss paid as an indemnity by the insurer, but we now assume that there is only a probability $\mathrm{q}, 0 < \mathrm{q} < 1$, that insurer can pay its claim, and that with probability $1 - \mathrm{q}$ the claim goes unpaid.[16] As a base case, we consider a fair premium, which we calculate taking the default risk into account as $\mathrm{P}(\alpha) = \alpha \mathrm{pqL}$.

Obviously such a premium is not realistic, since for q < 1 it implies that the insurer will default almost surely. More realistically the insurance will contain a premium loading of $\lambda > 0$. Thus $P(\alpha) = \alpha p[(1+\lambda)q]L$. Since P, α, p and L are known or observable, the consumer observes only $\mathrm{q}(1+\lambda)$, rather than q and λ separately. It is the consumer's *perception* of q and λ that will cause a deviation in insurance purchasing from the no-default-risk case. Since we only concern ourselves with how default risk affects insurance demand, the base case of a "fair premium" with $\lambda = 0$ seems like a good place to start.

Given our model, states of the world can be partitioned into three disjoint sets: states in which no loss occurs, states in which a loss occurs and the insurer pays its promised indemnity, and states in which a loss occurs but the insurer pays no indemnity. We assume that the individual's loss distribution is independent of the insurer's insolvency. Thus, the individual's objective can be written as

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[16] In a two-state (loss vs. no loss) model, there is no distinction between coinsurance and deductibles. A coinsurance rate α is identical to a deductible level of $\mathrm{D} = (1-\alpha)\mathrm{L}$.

(20) $$\underset{\alpha}{\text{maximize}}\ Eu = (1-p)u(Y_1) + pq\ u(Y_2) + p(1-q)u(Y_3)$$

where

$$Y_1 \equiv W - \alpha pqL$$

$$Y_2 \equiv W - \alpha pqL - L + \alpha L$$

$$Y_3 \equiv W - \alpha pqL - L$$

The first-order condition for maximizing (20) is

(21) $$\frac{dEu}{d\alpha} = -(1-p)pqLu'(Y_1) + pq(1-pq)Lu'(Y_2) - p(1-q)pqLu'(Y_3) = 0.$$

Dividing through by L and rearranging, we can rewrite (21) as

(22) $$u'(Y_2) = \beta u'(Y_1) + (1-\beta)u'(Y_3),$$

where $\beta = (1-p)/(1-pq),\ 0 < \beta < 1.$ Thus we see that $u'(Y_2)$ is a weighted average of $u'(Y_1)$ and $u'(Y_3)$.[17] Given the concavity of $u(\cdot)$, equation (22) implies that

(23) $$Y_1 > Y_2 > Y_3,$$

so that $\alpha^* < 1$. Clearly then, Mossin's Theorem does not hold in the presence of default risk.

In the presence of default risk, although we can purchase "nominally full insurance" with $\alpha^* = 1$, this does not fully insure the individual, since the insurer might not be able to pay a valid claim. Indeed, in the case where the insurer

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[17] Note that if there is no default risk with $q = 1$, then $u'(Y_1) = u'(Y_2)$ implying that $\alpha^* = 1$, as we already know from Mossin'sTheorem.

does not pay a filed claim, the individual is actually worse off than with no insurance, since the individual also loses his or her premium. The higher the level of insurance, the higher the potential loss of premium. Thus it is not surprising that $\alpha^* = 1$ is not optimal.

It also is not difficult to show that, in contrast to the case with no default risk, an increase in risk aversion will not necessarily lead to an increase in the level of insurance coverage. Although a more risk-averse individual would value the additional insurance coverage absent any default risk, higher risk aversion also makes the individual fear the worst-case outcome (a loss and an insolvent insurer) even more. More formally, let $\mathrm{v}(\cdot)$ be a more risk-averse utility function than $\mathrm{u}(\cdot)$. As in section 1.3, we know there exists an increasing concave function g, such that $\mathrm{v(y) = g[u(y)]}$ for all y.

Without losing generality, we can assume that $\mathrm{g'[u(Y_2)] = 1}$, so that $\mathrm{g'[u(Y_1)] < 1 < g'[u(Y_3)]}$. Now, calculating

$$(24) \quad \left.\frac{\mathrm{dEv}}{\mathrm{d}\alpha}\right|_{\alpha_{\mathrm{u}}^*} = -\mathrm{g'[u(Y_1)](1-p)pqLu'(Y_1) + pq(1-pq)Lu'(Y_2)} \\ - g'[u(Y_3)]p(1-q)pqLu'(Y_3).$$

Comparing (24) with (21), we see that one of the negative terms on the right-hand side in (24) is increased in absolute magnitude while the other is reduced. However, it is not possible to predetermine which of these two changes will dominate, *a priori*. Thus, we cannot predict whether α^* will increase or decrease.

Using similar arguments, it is easy to show that insurance is not necessarily an inferior good under DARA, as was the case without default risk. A somewhat more surprising result is that, under actuarially fair pricing, an increase in the probability of insolvency does not necessarily lead to a higher level of coverage. To see this, use the concavity of $\mathrm{Eu}(\mathrm{Y}(\alpha))$ in α, which is easy to check, and calculate

$$(25) \quad \left.\frac{\partial^2 Eu}{\partial\alpha\partial q}\right|_{\alpha^*} = p\alpha L[H(\alpha^*)] + p^2 qL[u'(Y_3) - u'(Y_2)],$$

where $\mathrm{H}(\alpha)$ is defined as the derivative in the first-order condition (21), with $\mathrm{u}(\mathrm{Y})$ replaced by the utility function $-\mathrm{u}'(\mathrm{Y})$. The level of insurance coverage will increase, due to an increase in q, if and only if (25) is positive. Although the second term on the right-hand side of (25) is positive, the first term can be either positive or negative. For example, if u exhibits DARA, it is straightforward to show that $-\mathrm{u}'$ is a more risk averse utility than u. Therefore, by our results on increases in risk aversion, $\mathrm{H}(\alpha^*)$ might be either positive or negative.

There are two, and only two, circumstances in which the form of the utility function u will yield $\mathrm{d}\alpha^*/\mathrm{dq} > 0$, regardless of the other parameters of the model (assuming fair prices). The first is where u is quadratic, so that $\mathrm{H}(\alpha) = 0$ for all α. The second is where u satisfies CARA, and which case $-\mathrm{u}'$ and u represent the same risk-averse preferences.[18] Hence, $\mathrm{H}(\alpha^*) = 0$. We also know for any

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[18] This is easiest to see by noting that $-\mathrm{u}'$ is an affine transformation of u.

risk-averse utility u, that $\mathrm{d}\alpha^*/\mathrm{dq} > 0$ for q sufficiently close to $\mathrm{q} = 1$. This follows since $\alpha^* = 1$ for $\mathrm{q} = 1$, but $\alpha^* < 1$ for $\mathrm{q} < 1$.

2.2 An Independent Background Risk

As opposed to a default risk, we now suppose that the insurer pays all of its claims, but that the individual's uninsured wealth prospect is $\mathrm{W} + \tilde{\varepsilon} - \tilde{\mathrm{x}}$, where $\tilde{\mathrm{x}}$ once again represents the insurable loss and where $\tilde{\varepsilon}$ represents a zero-mean background risk that is independent of $\tilde{\mathrm{x}}$. We assume that the support of the distribution of $\tilde{\varepsilon}$ is not the singleton $\{0\}$ and that $\mathrm{W} + \tilde{\varepsilon} - \tilde{\mathrm{x}} > 0$ almost surely. It is assumed that $\tilde{\varepsilon}$ cannot be insured directly. We wish to examine the effect of $\tilde{\varepsilon}$ on the choice of insurance level α^*.

The case of an independent background risk is easily handled by introducing the so-called *derived utility function* which we define as follows:

$$\text{(26)} \qquad \mathrm{v(y)} = \mathrm{Eu}(\mathrm{y} + \tilde{\varepsilon}) = \int_{-\infty}^{\infty} \mathrm{u}(\mathrm{y} + \varepsilon)\mathrm{dG}(\varepsilon),$$

where $\mathrm{G}(\cdot)$ is the distribution function for $\tilde{\varepsilon}$. Note that we can now write

$$\text{(27)} \qquad \max_{\alpha} Eu(\tilde{Y}(\alpha) + \tilde{\varepsilon}) = \int_0^L \int_{-\infty}^{\infty} u(Y(\alpha) + \varepsilon)dG(\varepsilon)dF(x) = \int_0^L v(Y(\alpha))dF(x)$$

$$= \mathrm{Ev}(\tilde{\mathrm{Y}}(\alpha)).$$

In other words, $\mathrm{v}(\mathrm{Y}(\alpha))$ is simply the "inner part" of an iterated integral. Finding the optimal insurance level for utility u in the presence of background risk $\tilde{\varepsilon}$, is identical to finding the optimal insurance level for utility v, absent any background risk.

For example, suppose u exhibits CARA or that u is quadratic. Then it is easy to show in each case that v is an affine transformation of u, so that background risk has no effect on the optimal choice of insurance.[19]

More generally, we know that more insurance will be purchased whenever the derived utility function $\mathrm{v}(\cdot)$ is more risk averse than $\mathrm{u}(\cdot)$. A sufficient condition for this to hold is *standard risk aversion* as defined by Kimball (1993). A utility function exhibits standard risk aversion "if every risk that has a negative interaction with a small reduction in wealth also has a negative interaction with any undesirable, independent risk." [Kimball (1993) p. 589] Here "negative interaction" means that risk magnifies the reduction in expected utility. Kimball shows that standard risk aversion is characterized by decreasing absolute risk aversion and decreasing absolute prudence, where absolute risk aversion is $\mathrm{r(y)} = -\mathrm{u''(y)/u'(y)}$ and absolute prudence is $\eta(y) = -u'''(y)/u''(y)$.

It is easy to show that DARA is equivalent to $\eta(y) > r(y)\ \forall y$. Since DARA implies prudence $(\text{i.e. } \mathrm{u'''(y)} > 0)$, then under DARA the function $-\mathrm{u'(y)}$ represents a risk-averse utility of its own. The condition $\eta(\mathrm{y}) > \mathrm{r(y)}$ thus implies that $-\mathrm{u}'(\cdot)$ is a more risk-averse utility than $\mathrm{u}(\cdot)$. Similarly, we find that decreasing absolute prudence or "DAP" implies that $u''''(y) < 0$ and that $\mathrm{u}''(\cdot)$ is a more risk-averse utility function than $-\mathrm{u}'(\cdot)$.

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[19] For CARA, $\mathrm{v(y) = ku(y)}$ and for quadratic utility $\mathrm{v(y) = u(y) + c}$, where $k = E[\exp\ (r\tilde{\varepsilon})] > 0$ and $\mathrm{c} = -\mathrm{t}\,\mathrm{var}(\tilde{\varepsilon})$ for some $\mathrm{t} > 0$. Gollier and Schlesinger (1998) show that these are the only two forms of u for which v represents preferences identical to u.

Let $\pi(\mathrm{y})$ denote the risk premium, as defined by Pratt (1964), for utility $\mathrm{u}(\cdot)$, given base wealth y and fixed risk $\tilde{\varepsilon}$. Similarly, let $\pi_1(\mathrm{y})$ and $\pi_2(\mathrm{y})$ denote the corresponding risk premia for utilities $-\mathrm{u}'(\cdot)$ and $\mathrm{u}''(\cdot)$ respectively. That is,

$$\begin{aligned} Eu(y+\tilde{\varepsilon}) &= u(y-\pi(y)) \\ -Eu'(y+\tilde{\varepsilon}) &= -u'(y-\pi_1(y)) \\ Eu''(y+\tilde{\varepsilon}) &= u''(y-\pi_2(y)). \end{aligned} \tag{28}$$

Standard risk aversion thus implies that $\pi_2(\mathrm{y}) > \pi_1(\mathrm{y}) > \pi(\mathrm{y}) > 0 \ \forall \mathrm{y}$. Thus, we have the following set of inequalities

$$-\frac{\mathrm{v}''(\mathrm{y})}{\mathrm{v}'(\mathrm{y})} = \frac{-\mathrm{Eu}''(\mathrm{y}+\tilde{\varepsilon})}{\mathrm{Eu}'(\mathrm{y}+\tilde{\varepsilon})} = \frac{-\mathrm{u}''(\mathrm{y}-\pi_2)}{\mathrm{u}'(\mathrm{y}-\pi_1)} > \frac{-\mathrm{u}''(\mathrm{y}-\pi_1)}{\mathrm{u}'(\mathrm{y}-\pi_1)} > \frac{-\mathrm{u}''(\mathrm{y})}{\mathrm{u}'(\mathrm{y})}, \tag{29}$$

where the last inequality follows from DARA. Consequently $\mathrm{v}(\cdot)$ is more risk-averse than $\mathrm{u}(\cdot)$.[20]

Considering the maximization program (29), the above result taken together with our previous results on increases in risk aversion, implies the following:

Proposition 4: *(a) If insurance has a zero premium loading, $\lambda = 0$, then full coverage is optimal in the presence of an independent background risk. (b) If insurance premia include a positive loading, $\lambda > 0$, then partial coverage is optimal in the presence of an independent background risk. (c) If insurance premia include a positive loading, $\lambda > 0$ and utility exhibits standard risk aversion,*

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[20] Another simple proof that standard risk aversion is sufficient for the derived utility function to be more risk averse appears in Eeckhoudt and Kimball (1992). Standard risk aversion is stronger than necessary, however. See Gollier and Pratt (1996).

then more coverage is purchased in the presence of an independent zero mean background risk.

Remark: Parts (a) and (b) above do not require $E\tilde{\varepsilon} = 0$. They are direct applications of Mossin's Theorem to utility $v(\cdot)$. Although the discussion above is for proportional coinsurance, part (c) of Proposition 4 also applies to deductibles, since it only relies upon $v(\cdot)$ being more risk-averse than $u(\cdot)$.

2.3 Nonindependent Background Risk

Obviously the background risk need not always be statistically independent of the loss distribution. For example, if $\tilde{\varepsilon} = \tilde{x}$ then final wealth is risk free without insurance, Y=W. Buying insurance on $\tilde{x}$ would only introduce risk into the individual's final wealth prospect. Consequently, zero coverage is optimal, even at a fair price, $\lambda = 0$. For example, suppose the individual's employer provides full insurance coverage against loss $\tilde{x}$. We can represent this protection by $\tilde{\varepsilon}$ as described here; and thus no further insurance coverage would be purchased.

Similarly, if $\tilde{\varepsilon} = -\tilde{x}$ then final wealth can be written as $\tilde{Y} = W - 2\tilde{x}$ with no insurance. Treating $2\tilde{x}$ as the loss variable, Mossin's Theorem implies that full insurance on $2\tilde{x}$ will be optimal at a fair price. This can be achieved by purchasing insurance with a coinsurance level of $\alpha^* = 2$. Although this is nominally "200% coverage," it is defacto merely full coverage of $2\tilde{x}$. If insurance is constrained to exclude overinsurance, then $\alpha = 1$ will be the constrained optimum. For insurance markets with a premium loading $\lambda > 0$, Mossin's

Theorem implies that $\alpha^* < 2$. In this case, a constraint of no overinsurance might or might not be binding.

For more general cases of nonindependent background risk, it becomes difficult to predict the effects on insurance purchasing. Part of the problem is that there is no general measure of dependency that will lead to unambiguous effects on insurance demand. Correlation is not sufficient since other aspects of the distributions of $\tilde{x}$ and $\tilde{\varepsilon}$, such as higher moments, also are important in consumer choice.[21] Alternatives measures of dependence, may based on stochastic dominance, do not lead to definitive qualitative effects on the level of insurance demand.

For example, suppose we define the random variable $\tilde{\varepsilon}'$ to have the same marginal distribution as $\tilde{\varepsilon}$, but with $\tilde{\varepsilon}'$ statistically independent of $\tilde{x}$. We can define a partial stochastic ordering for $W + \tilde{\varepsilon} - \tilde{x}$ versus $W + \tilde{\varepsilon}' - \tilde{x}$. If, for example, we use second-degree stochastic dominance, we will be able to say whether or not the risk-averse consumer is better off or worse off with $\tilde{\varepsilon}$ or $\tilde{\varepsilon}'$ as the source of background risk; but we will not be able to say whether the level of insurance demanded will be higher or lower in the presence of background risk $\tilde{\varepsilon}$ versus background risk $\tilde{\varepsilon}'$.

Some recent work has used more sophisticated partial orderings to examine the behavior of insurance demand in the presence of a background risk that is not statistically independent from the loss distribution. For the most part,

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[21] Doherty and Schlesinger (1983b) use correlation, but restrict the joint distribution of $\tilde{x}$ and $\tilde{\varepsilon}$ to be bivariate normal. For other joint distributions, correlation is not sufficient.

this work has focussed on comparing insurance demands with and without the background risk.[22] Eeckhoudt and Kimball (1992), for example, use one particular partial ordering, assuming that the conditional distribution of $\tilde{\varepsilon}$ given x_1 dominates the conditional distribution of $\tilde{\varepsilon}$ given x_2 via third-degree stochastic dominance, for every $x_1 < x_2$. Eeckhoudt and Kimball go on to show that such a negative dependency between $\tilde{\varepsilon}$ and $\tilde{x}$ leads to an increase in insurance demand in the presence of background risk, whenever preferences exhibit standard risk aversion. Important to note here, is that even with the strong third-degree stochastic dominance assumption, risk aversion alone is not strong enough to yield deterministic comparative statics.

One paper that does compare insurance demands for a change in background risk from $\tilde{\varepsilon}'$ to $\tilde{\varepsilon}$, where $\tilde{\varepsilon}'$ is statistically independent from $\tilde{x}$ and has the same marginal distribution as $\tilde{\varepsilon}$, is Tibiletti (1995). She was the concept of concordance as her partial ordering. In particular, if $H(\varepsilon, x)$ is the joint distribution of the random vector $(\tilde{\varepsilon}, \tilde{x})$ and $G(\varepsilon, x)$ the distribution of $(\tilde{\varepsilon}', \tilde{x})$, then H is *less concordant* then G if $H(\varepsilon, x) \geq G(\varepsilon, x) \ \ \forall \varepsilon, x$. In other words, G dominates H by joint first –degree stochastic dominance. However, even using concordance, we need to make fairly restrictive assumptions on preferences to yield deterministic comparisons between optimal levels of insurance purchases. In particular, suppose we restrict the degree of relative prudence, $y\eta(y) = -yu'''(y) / u''(y)$, to be no greater than on. Then for H less concordant

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[22] Aboudi and Thon (1995) do an excellent and thorough job of characterizing many of the potential partial orderings, albeit in a discrete probability space, but hey only whet our appetite for applying these ordering to insurance demand.

than G, more insurance will be purchased under H; i.e., more insurance is purchased in the presence of background risk $\tilde{\varepsilon}$ than in the presence of the independent background risk $\tilde{\varepsilon}'$,

Note that concordance is yet another measure of positive dependency between $\tilde{\varepsilon}$ and $\tilde{x}$. Thus the above result implies that if $\tilde{\varepsilon}$ and $\tilde{x}$ are, in a certain sense, negatively associated with each other, so that higher losses are more readily exacerbated by the simultaneous realization of low background wealth, then more insurance is purchased. In other words, the individual can partly compensate for downward fluctuations in background risk $\tilde{\varepsilon}$ by increasing his protection on the insurable loss $\tilde{x}$. While this result seems intuitively appealing, note that Tibiletti's result above, just as the result of Eeckhoudt and Kimball (1992), does *not* automatically follow if we assume only risk aversion for consumer preferences. In particular, if we assume that we change from zero background risk to a background risk that is negatively associated with $\tilde{\varepsilon}$ (either as measured by concordance, or as by Eeckhoudt and Kimball, 1992), there exist examples of risk-averse utility functions that would lead to the counter-intuitive result that insurance demand is lower in the presence of the background risk.[23]

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[23] Although results are sparse and restrictive, this seems to be an area of much recent research activity. Tibiletti (1995) introduces the use of *copulas*, which allow one to write the joint distribution of ($\tilde{\varepsilon}$, $\tilde{x}$) as another joint distribution function of the marginal distributions of $\tilde{\varepsilon}$ and $\tilde{x}$, to analyze this problem. The use of particular functional forms for the *copulas* allows one to parameterize the degree of statistical association between $\tilde{x}$ and $\tilde{\varepsilon}$. See Frees and Valdez (1998) for a survey of the current use of *copulas*. The fact that a detrimental change in the background risk $\tilde{\varepsilon}$ does not necessarily lead to higher insurance purchases is examined by Eeckhoudt, Gollier and Schlesinger (1996), for the case where the deterioration can be measured by first- or second-degree stochastic dominance.

3. Concluding Remarks

Mossin's Theorem is often considered to be the cornerstone result of modern insurance economics. Indeed this result depends only on risk aversion for smooth preferences, such as those found in the expected-utility model.[24]

Although many results depend on stronger assumptions than risk aversion alone, research has turned in this direction. Stronger measures of risk aversion, such as those of Ross (1981) and of Kimball (1993), have helped in our understanding more about the insurance-purchasing decision.

One common "complaint," that I hear quite often from other academics, is that these restrictions on preferences beyond risk aversion are too limiting. These critics might be correct, if our goal is to guess at reasonable preferences and then see what theory predicts. However, insurance demand is not just a theory. I doubt there is anyone reading this who does not possess several insurance policies. If our goal in setting up simple theoretical models is to capture behavior in a positive sense, then such restrictions on preferences might be necessary. Of course, one can always argue that more restrictions belong elsewhere in our models, not on preferences.

As mentioned previously, the single-risk model as presented here should be viewed as a base case. As new insights about preferences become known,

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[24] Actually, this result hangs on the differentiability of the vonNeuman-Morgenstern utility function. "Kinks" in the utility function can lead to violations of Mossin's result. See, for example, Eeckhoudt, Gollier and Schlesinger (1997).

this model should extend in many ways. Indeed, many extensions already are to be found in this volume. Certainly there are enough current variations in the model so that every reader should find something of interest. I look forward to seeing the directions in which the theory of insurance demand is expanded in the years to come.

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