EC 413
Economic Forecast and Analysis
(Professor Lee)

Lecture 2

Moving Average and Exponential Smoothing

Read:

(WK Ch 3)

This lecture is about:

Simple short-run forecasting tools based on some underlying pattern to the data

- *Smoothed* curve (eliminate up-and-down movement)
- Trend
- Seasonality

1. (Simple) Moving Averages

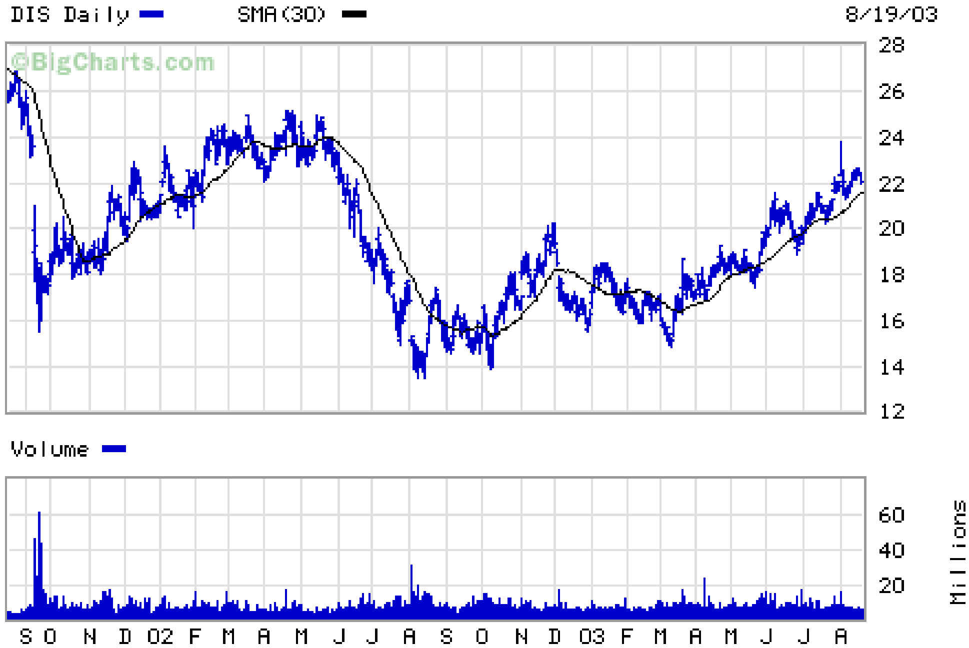
(Ex 1) 3 periods moving averages

$$\tilde{y}_t = \frac{1}{3}(y_{t-1} + y_{t-2} + y_{t-3})$$

Also, 5 periods MA can be considered.

Period	Actual	3 Quarter MA Forecast	5 Quarter MA forecast
Mar-83	**239.3**	Missing	Missing
Jun-83	**239.8**	Missing	Missing
Sep-83	**236.1**	Missing	Missing
Dec-83	232	**238.40**	Missing
Mar-84	224.75	235.97	Missing
Jun-84	237.45	230.95	**234.39**
Sep-84	245.4	231.40	234.02
Dec-84	251.58	235.87	235.14
…		So on..	

(Ex 2) Disney Stock, 30 days MA



Notes:

(i) One can impose *weights* and use weighted moving averages (WMA).

eg) $\tilde{y}_t = 0.25\, y_{t-1} + 0.5\, y_t + 0.25\, y_{t+1}$

(ii) How many periods to use is a question; more significant smoothing-out effect with longer lags.

(iii) Peaks and troughs (bottoms) are not predicted.

(iv) Events are being averaged out.

(v) Since any moving average is serially correlated, any sequence of random numbers could appear to exhibit cyclical fluctuation.

2. Simple Exponential Smoothing

Concepts:

- Also, supressing short-run fluctuation by smoothing the series
- Weighted averages of all previous values with more weights on recent values
- No trend, No seasonality

Model:

$$\tilde{y}_t = \alpha\, y_{t-1} + (1 - \alpha)\, \tilde{y}_{t-1}$$

or

$$\tilde{y}_t = \tilde{y}_{t-1} + \alpha\, (y_{t-1} - \tilde{y}_{t-1})$$

where $(y_{t-1} - \tilde{y}_{t-1})$ is ‘forecast error’.

Remarks on a (smoothing parameter).

(i) Normally, choose α between 0 and 1.

(ii) If $\alpha = 1$, it becomes a naïve model; if α is close to 1, more weights are put on recent values. The model fully utilizes forecast errors.

(iii) If α is close to 0, distant values are given weights comparable to recent values. Choose α close to 0 when there are big random variation in the data.

(iv) α is often selected as to minimize the MSE.

Forecast:

$\tilde{y}_{T+h} = \tilde{y}_T$ for all h > 0. (T = end period)

Why exponential?

$$\tilde{y}_t = \alpha\, y_{t-1} + (1-\alpha)\, \boxed{\tilde{\mathbf{y}}_{\mathbf{t-1}}}$$

$$\boxed{\tilde{\mathbf{y}}_{\mathbf{t-1}}} = \alpha\, y_{t-2} + (1-\alpha)\, \boxed{\tilde{y}_{t-2}}$$

$$\boxed{\tilde{\mathbf{y}}_{\mathbf{t-2}}} = \alpha\, y_{t-3} + (1-\alpha)\, \boxed{\tilde{y}_{t-3}}$$

$$= \alpha\, y_{t-1} + (1-\alpha)\alpha y_{t-2} + (1-\alpha)\alpha^2 y_{t-3} + \ldots + (1-\alpha)\, \boxed{a^{\mathbf{k}}}\, y_{t-k+1}$$

$\boxed{a^{\mathbf{k}}}$ decreases *exponentially*.

Example: Consumer Sentiment

Data: consumer_sentiment_SEM.xls

Result: α is estimated as 0.6.

RMSE = 2.65

Plot: No trend, no seasonality.

3. Holt's Exponential Smoothing

Concepts:

- Introduce a Trend factor to the simple exponential smoothing method
- Trend, but still No seasonality

Model:

$$\tilde{y}_t = \alpha\, y_{t-1} + (1 - \alpha)\,(\, \tilde{y}_{t-1} + \widetilde{T}_{t-1}\,)$$

$$\widetilde{T}_t = \gamma\,(\tilde{y}_t - \tilde{y}_{t-1}) + (1 - \gamma)\,\widetilde{T}_{t-1}$$

where $(\tilde{y}_t - \tilde{y}_{t-1})$ captures *trend*.

Two parameters :

α = smoothing parameter

γ = trend coefficient

; One can impose a priori values or these can be determined from the data.

Forecast:

$$\tilde{y}_{T+h} = \tilde{y}_T + h\tilde{T}_T$$

.. Trend prediction is added in the h-step ahead forecast.

Example: SP 500 series

Data: sp500_holt.xls
Result: Estimate for $\alpha = 0.64$;
Estimate for $\gamma = 0.24$.
RMSE = 66.95
Plot: Clearly trend, but no seasonality

4. Holt-Winter's Exponential Smoothing

Concepts:

- Introduce both Trend and seasonality factors
- Seasonality can be added additively or multiplicatively.

Model (multiplicative):

$$\tilde{y}_t = \alpha\, y_t / \tilde{S}_{t-p} + (1-\alpha)\,(\tilde{y}_{t-1} + \tilde{T}_{t-1})$$

$$\tilde{T}_t = \gamma\,(\tilde{y}_t - \tilde{y}_{t-1}) + (1-\gamma)\,\tilde{T}_{t-1} \qquad \text{(trend)}$$

$$\tilde{S}_t = \beta\,(y_t / \tilde{y}_t) + (1-\beta)\,\tilde{S}_{t-p} \qquad \text{(seasonality)}$$

where $(y_t / \tilde{y}_t)$ captures *seasonal effects*.
p = # of periods in the seasonal cycles
(p = 4, for quarterly data)

Three parameters :
α = smoothing parameter
γ = trend coefficient
β = seasonality coefficient

Forecast:

$$\tilde{y}_{T+h} = [\,\tilde{y}_T + h\tilde{T}_T\,]\,\tilde{S}_{T+h-p}$$

.. Seasonal factor is multiplied in the h-step ahead forecast.

Example: Truck production

Data: truck_prod_holt_winter.xls

Result: Estimate for α = 0.41;
Estimate for γ = 0.03;
Estimate for β = 0.37.

RMSE = 30.05

Plot: Clearly trend, clearly seasonality

Summary: When can each of the following techniques be used?

- Moving Average Models
- Simple Exponential Smoothing
- Holt's Exponential Smoothing
- Holt-Winter's Exponential Smoothing

Review Questions

1. Discuss how one can choose α values. How would these different α values weigh past observations of the variable to be forecast? If $\alpha = 0.9$ provided the best forecast for your data, what would this imply?

2. Does exponential smoothing place more or less weight on the most recent data when compared with the moving average model? What weight is applied to remote values?

3. Why is smoothing (simple, Holt's and Winter's) also called *exponential smoothing?*

4. (WK Ex 3.7, 3.11, p. 135-136) Consider the data on mobile-home shipments (data: mobile_home.xls) over the period from 1986 Q.1 to 1995 Q.4. The data are in thousands of units.

   (a) Calculate both the three quarter and five quarter moving average for these data and compare the forecasts by calculating the RMSE.
   (b) Plot the data and (eye) examine the existence of trend and seasonality in the data. Which smoothing method can you recommend?
   (c) Apply three exponential smoothing methods (simple, Holt and Winter's) to the data, and compare the RMSE. Make sure to report the estimated parameter values in each model.
   (d) Obtain the out-of-sample forecasts for each of the three models on the four quarters of 1996. Then, calculate the RMSE of the out-of-sample forecasts using the following actual values.

1996 Q.1	84.4
1996 Q.2	97.2
1996 Q.3	94.9
1996 Q.4	86.9