

Self-test (Lecture 1)

1. List all classical assumptions and the corresponding problems when each of them is not satisfied what are the remedy for each problem?

2. List the sources of endogeneity what is the definition of endogeneity?

3. Let a & b are $p \times$ vectors. Prove $\frac{\partial a'b}{\partial a} = b$, $\frac{\partial a'b}{\partial b} = a$

4. Find $\hat{\beta}$ that minimizes $e'e$ where $y = X\beta + e$

5. Find $\hat{\alpha}$ and $\hat{\beta}$ that minimize $e'e$ where $y = \alpha + X\beta + e$
(we need two F.O.C: $de'e/d\alpha = 0$, $de'e/d\beta = 0$)

6. Show that a) $X'\hat{e} = 0$, $\hat{y}'\hat{e} = 0$, $MX = 0$, $MP = 0$, $PX = X$, $MX_1 = 0$
 $PX_1 = X_1$ where $X = (X_1, X_2)$, M & P matrices are idempotent matrices as defined in the note.

b) $E(\hat{\beta}) = \beta$, $\hat{\beta} = \beta + (X'X)^{-1}X'e$

c) $Var(\hat{\beta}) = \hat{\sigma}^2(X'X)^{-1}$ or $(X'X)^{-1}(\sum \hat{e}_i^2 X_i'X_i)(X'X)^{-1}$
under which condition?

d) Let $Var(\hat{\beta}) = \sigma^2(X'X)^{-1}$. Find $Var(R\hat{\beta} - r)$, $Var(\sqrt{n}\hat{\beta})$, $Var(\hat{\beta})$ where $\hat{\beta} = [(X'X)^{-1}X'D]$, $Var(n\hat{\beta} - \beta_0)$, $Var(X'\hat{\beta})$

e) Let $\tilde{e} = y - X\tilde{\beta}$, $\hat{e} = y - X\hat{\beta}$. Show that
 $\tilde{e}'\tilde{e} - \hat{e}'\hat{e} = (\tilde{\beta} - \hat{\beta})'X'X(\tilde{\beta} - \hat{\beta})$

f) $\hat{e}'\hat{e} = y'My = e'Me$

7. Let (1) $y = x_1\beta_1 + x_2\beta_2 + e$ a) show that $\hat{\beta}_2 = \hat{b}_2$, $\hat{b}_1 = \hat{b}_1^*$
(2) $y = x_1b_1 + \tilde{x}_2b_2 + u$ b) Suppose that (1) is a true model, but one uses (2). Find the bias of \hat{b}_1^* and discuss the direction of bias.
(3) $y = x_1b_1^* + \varepsilon$ c) Suppose that (3) is a true model, but one uses (1). Show that $\hat{\beta}_1$ is unbiased. Show that $E(\hat{\beta}_1) = 0$.

8. Let $\hat{\sigma}^2 = \frac{1}{n-k} \hat{e}'\hat{e}$ show that a) $\frac{(n-k)\hat{\sigma}^2}{\sigma^2} \sim \chi_{n-k}^2$. [use the fact $E(\chi_{n-k}^2) = n-k$, why?]
b) $E(\hat{\sigma}^2) = \sigma^2$

9. Let $\hat{\sigma}^2 (X'X)^{-1} = \begin{pmatrix} 3.5 & -1.5 \\ -1.5 & 0.75 \end{pmatrix}$ where $y = x_1\beta_1 + x_2\beta_2 + e$
 $\hat{y} = 3x_1 + 0.5x_2$

Find the t-statistic of

- i) $H_0: \beta_1 = 0$
- ii) $H_0: 2\beta_1 - 3\beta_2 = 2$
- iii) $H_0: \beta_1 - 2\beta_2 = 1, \beta_2 = 2$
- iv) $H_0: \beta_1 = \beta_2$

10. Show that $\frac{R\hat{\beta} - r}{SR} \sim t_{n-k}$ where $SR = s^2 R(X'X)^{-1}R'$
 $s^2 = \frac{1}{n-k} \hat{e}'\hat{e}$

11. Find $\hat{\beta}$ that minimizes $\hat{e}'\hat{e}$ subject to $R\beta = r$
 where $\hat{e} = y - X\hat{\beta}$

12. Let

x_2	y_i
1	5
	3
	4

 $y = a + bx_2 + e_i$ $(X'X)^{-1} = \begin{pmatrix} 2.33 & -1 \\ -1 & 0.5 \end{pmatrix}$

- a) Find $\hat{a}, \hat{b}, \hat{e}_i, \hat{\sigma}^2$
- b) Find the t-stat for $b=1$
- c) Find Wald, F and LM stat. for $H_0: b=1$, using $\hat{\sigma}^2 (X'X)^{-1}$.

13. What is a HAC variance? What is a HC variance?

14. Let $\log \text{wage} = 1.0 + 0.15 \text{Edu} - 0.25 \text{female}$
 with $\hat{r} = 0.45$ " \int_0^1 female $\frac{\text{female}}{y/w}$ "

- a) Precisely interpret the coeff. of Edu.
- b) " " female
- c) What is a precise predicted wage of a female worker with 16 yrs of edu?

15. Describe the procedure for the RESET test.

16. Let $y = x_1\beta_1 + x_2\beta_2 + e$ and obtain $\hat{\beta}_1$ as an OLS estimator
 let $y = x_1b_1 + u$ and obtain \tilde{b}_1 as an OLS estimator.
 Show: $\text{Var}(\tilde{b}_1 - \hat{\beta}_1) = \text{Var}(\tilde{b}_1) - \text{Var}(\hat{\beta}_1)$