GEE Models

POLS571: Longitudinal Data Analysis
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GLMs

- Models in which one specifies the "link" function $E(Y_i) \equiv \mu_i = g(X_i\beta)$ and the relationship between the mean and the variance (e.g. $V_i = \frac{g(\mu_i)}{\phi}$).

- Standard GLMs obtain estimates by solving the "score equations":

$$U_k(\beta) = \sum_{i=1}^{N} D_i V_i^{-1}(Y_i - \mu_i) = 0 \qquad (1)$$

  where $D_i = \frac{\partial \mu_i}{\partial \beta}$ and V_i is the variance matrix, above.

- See e.g. McCullagh and Nelder 1989; Gill 2000.

GLMs for Correlated Data

- Consider $E(Y_{it}) \equiv \mu_{it} = g(X_{it}\beta)$, $T > 1$.

- Must make some provision for dependence within i, across t.

The answer: Specify the conditional within-unit correlation...

- Define the "working" $T \times T$ correlation matrix $\mathbf{R}_i(\alpha)$ as a function of α.

- Structure of $\mathbf{R}_i(\alpha)$ determined by the investigator.

- Then redefine the variance matrix in (1) as:

$$V_i = \frac{(A_i)^{\frac{1}{2}}\mathbf{R}_i(\alpha)(A_i)^{\frac{1}{2}}}{\phi} \tag{2}$$

- Intuition:

  1. Choose β so that μ_{it} is "close" to Y_{it} on average,

  2. Optimally weight each residual $(Y_{it} - \mu_{it})$ by the inverse of $Cov(Y_i)$.

Correlation Structures

- "Working Independence" : $\mathbf{R}_i(\alpha) = \mathbf{I}$
  - No within-unit correlation.
  - Equivalent to (e.g.) standard logit/probit.
- "Exchangeable" : $\mathbf{R}_i(\alpha) = \rho$
  - Observations covary equally within units.
  - Estimate a single parameter.
  - Similar to "random effects".

- Autoregressive : $\mathbf{R}_i(\alpha) = \rho^{|t-s|}$
  - Here, AR(1).
  - Correlation decays over (e.g.) time.
  - Can also do "banded" / "stationary" correlations.

- "Unstructured" : $\mathbf{R}_i(\alpha) = \alpha_{st}, t \neq s$
  - α is a $T \times T$ matrix.
  - Estimate $\frac{T(T-1)}{2}$ unique pairwise correlations
  - Very flexible; often hard to estimate.

“Robust” Standard Errors

- $\widehat{\beta}_{GEE}$ is robust to misspecification of $\mathbf{R}_i(\alpha)$

- Var($\widehat{\beta}_{GEE}$) is not.

- Solution: “sandwich” estimator:

$$\begin{aligned} \widehat{Var(\widehat{\beta}_{GEE})} &= N(\sum_{i=1}^{N} \widehat{D}_i' \widehat{V}_i^{-1} \widehat{D}_i)^{-1} \\ &\quad (\sum_{i=1}^{N} \widehat{D}_i' \widehat{V}_i^{-1} S_i \widehat{V}_i^{-1} \widehat{D}_i)(\sum_{i=1}^{N} \widehat{D}_i' \widehat{V}_i^{-1} \widehat{D}_i)^{-1} \end{aligned}$$

where $S_i = (Y_i - \mu_i)(Y_i - \mu_i)'$.

- Similar to the Huber/White estimator – robust to misspecification.

GEE2

- Consider the $m = \frac{T(T-1)}{2}$ elements of $\mathbf{R}_i(\alpha)$.
  - Normally "nuisance parameters".
  - May wish to estimate them as well.

- Can do this with a separate estimating equation:

$$U_m(\alpha) = \sum_{i=1}^{N} E_i' W_i^{-1}(Z_i - \eta_i) \qquad (3)$$

- Can be estimated either separately from $U_k(\beta)$ [with $Cov(U_k(\beta), U_m(\alpha)) = 0$], or allowing the two to covary.

- Drawbacks: Requires correct specification of $\mathbf{R}_i(\alpha)$ for consistent estimates of $\widehat{\beta}$.

Software

- Stata (`-xtgee-`, my personal favorite)
- SAS
- S-Plus / R
- GAUSS
- Specialized packages (esp. for GEE2)

Example: Supreme Court Dissents

- Outcome variable: Number of dissenting opinions, by justice and year, 1789-1994.

- $N \approx 108, T \approx 206$.

- Model:

$$\begin{aligned} DISSENTS_{jt} &= f[\beta_0 + \beta_1 ln(CASES_t) + \\ &\quad \beta_2 CHIEF_{jt} + \beta_3 FRESHMAN_{jt} + \\ &\quad \beta_4 TENURE_{jt} + \beta_5 NORM_t + \\ &\quad \beta_6 MINORITY_{jt} + u_{jt}] \end{aligned}$$

- Compare GEE (Stata's `-xtpois, pa-`) and random-effects Poisson (Stata's `-xtpois, re-`) models.

Model Comparisons

Variable	Random Effects	GEE Exchangeable	GEE AR(1)
(Constant)	-3.52	-3.59	-3.75
	(0.31)	(0.72)	(0.54)
ln(Cases)	0.78	0.71	0.73
	(0.06)	(0.12)	(0.09)
Chief Justice	-1.20	-1.35	-0.92
	(0.11)	(0.22)	(0.19)
Freshman	-1.00	-1.03	-0.95
	(0.10)	(0.25)	(0.20)
Tenure	0.03	0.03	0.03
	(0.001)	(0.01)	(0.01)
Consensual Norm	1.40	2.18	2.30
	(0.12)	(0.22)	(0.15)
Minority Justice	0.16	0.17	0.15
	(0.04)	(0.15)	(0.16)
$\widehat{\alpha}$	0.69		
	(0.12)		
$\widehat{\rho}$		0.27	0.53
NT	1730	1730	1728

Conclusions

- GEEs useful when correlation isn't the central focus (and even when it is...).

- Can be used with continuous, binary, count response variables.

- Available in most widely-used software packages.