- More on log-linear models

$$\log y = a + b_1 X_1 + b_2 D + e \qquad \text{— (1)}$$

X_1 = continuous
D = dummy var

i) partial effect : $100\%(e^{\hat{b}_i} - 1)$

Roughly, $\hat{b}_1$ = % change of y when X_1 increases by 1 unit

precisely, % change of y is given as

$$100 \times (e^{\hat{b}_1} - 1)\,\%$$

why?

$$\%\text{ change of } y = 100\% \times \frac{\Delta Y}{Y} = 100\% \times \frac{Y_{New} - Y_{old}}{Y_{old}}$$

$$= 100\%\left(\frac{Y_{new}}{Y_{old}} - 1\right) = 100\%\left[\frac{e^{\hat{a} + \hat{b}_1(X_1+1) + \hat{b}_2 D}}{e^{\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 D}} - 1\right]$$

$$= 100\%(e^{\hat{b}_1} - 1)$$

Also, $\hat{b}_2$ = % difference of y between two groups

$$\%\text{ difference} = 100\%\left(\frac{Y_1 - Y_0}{Y_0}\right) = 100\%\left(\frac{Y_1}{Y_0} - 1\right)$$

$$= 100\%\left(\frac{e^{\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 \cdot 1}}{e^{\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 \cdot 0}} - 1\right) = 100\%(e^{\hat{b}_2} - 1)$$

eg) If $\hat{b}_1 = 0.306$

$$\%\text{ change of } y = 100\% \times (e^{0.306} - 1) = 35.8\%$$

If $\hat{b}_2 = -0.052$

$$\%\text{ change of } y = 100\% \times (e^{-0.052} - 1) = -5.1\%$$

ii) prediction ; we're interested in y.

$$\hat{y} = \exp(\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 D) \cdot \underbrace{\exp(\hat{\sigma}^2/2)}$$

precisely, this term needs to be multiplied

point: y has a log-normal dist.

(log y has a normal dist : $u \sim N(0, \sigma^2)$, $E(u) = 0$)

H/W
let $u \sim N(0, \sigma^2)$
(a) Find the density of $W = e^u$.
(b) Find $E(W)$.

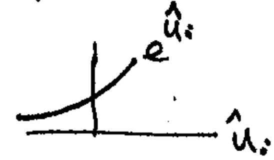
$E(e^u) \neq 0 \Rightarrow E(e^u) = \exp(\sigma^2/2)$

thus, the above result is based on the normality assumption.

Alternatively (seldom used, but see Wooldridge, p 211)

$$\hat{y} = \exp(\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 D) \cdot \hat{c} = \exp(\widehat{\log y}) \cdot \hat{c}$$

(a) $\hat{c} = \frac{1}{N} \sum \exp(\hat{u}_i)$ where $\hat{u}_i$ = residuals from (1)



Note $\hat{c} > 1$ always

; method of moments

(b) $\hat{c}$ = coefficient of the regression of y on $\exp(\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 D)$

$$y = \exp(\widehat{\log y}) \cdot c + \text{error}$$

(without a constant term)

Obtain $\hat{c}$ from this regression.

H/W Use the data set wages1

a) Find the predicted wage of a male worker who has 10 yrs of expr and 13 yrs of school. Use 3 methods

b) Find the precise estimates of partial effects.

– More on marginal effects and predictions
(nonlinear models ; interaction terms)

$$y = f(X, \beta) + e$$

count data models

eg) $y_i = \exp(X_i\beta) + e_i$ or $y_i = \exp(X_i\beta) \cdot \exp(u_i)$
$\Rightarrow \ln y_i = X_i\beta + u_i$

$y_i = \dfrac{1}{1 + \exp(-X_i\beta)} + e_i \Rightarrow$ this is a logit model
cdf of the logistic distribution

$y_i = F(X_i\beta) + e_i$
where $F(\cdot)$ is the cdf of the std. normal dist.

... many examples

Ex1) $y = \exp(a + b_1 X_1 + b_2 D) + e$

- $\widehat{\dfrac{dy}{dX_1}} = \exp(\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 D) \cdot \hat{b}_1 \neq \hat{b}_1$
  ... depends on $\hat{b}_1$, $(\hat{a}, \hat{b}_1, \hat{b}_2)$, and X_{1i}, D_i
  .. varies over different observations (i)

- differences of two groups ($D = 1, 0$)
  $= \exp(\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 \cdot 1) - \exp(\hat{a} + \hat{b}_1 X_1 + \hat{b}_2 \cdot 0)$
  $\neq \hat{b}_1$

  point: partial effects of dummy variables should be obtained by the difference of predicted values.

Ex2) $y = \exp(a + b_1 X_1 + b_2 X_2 + b_3 X_1 X_2) + e$

- Interaction effect: $\dfrac{d^2 y}{dX_1 dX_2} \neq \hat{b}_3$

  point: The t-static of b_3 does not provide the significance of the interaction effect.

$$\frac{dy}{dX_1} = \exp(\cdot)(\hat{b}_1 + \hat{b}_3 X_2)$$

$$\frac{d^2y}{dX_1 dX_2} = \exp(\cdot)(\hat{b}_3) + \underbrace{\exp(\cdot)(\hat{b}_1 + \hat{b}_3 X_2)(\hat{b}_3)}_{\text{this cannot be ignored}} \neq \exp(\cdot)\hat{b}_3$$

ex3) Similarly, in probit & logit models

$$P = \frac{1}{1+\exp(-a-b_1X_1-b_2X_2-b_3(X_1X_2))} \text{ or } F(a+b_1X_1+b_2X_2+b_3X_1X_2)$$

$$\frac{dP}{dX_1} = F(\cdot)\cdot\hat{b}_1$$; depends on $\hat{b}_1$, $\hat{a}$, $\hat{b}_2$ and X_{1i}, D_i

$$\frac{d^2P}{dX_1 dX_2} \neq F(\cdot)\, b_3$$

- differences of two groups (X_1, D are regressors)

$$= F(\hat{a}+\hat{b}_1X_1+\hat{b}_2\cdot 1) - F(\hat{a}+\hat{b}_1X_1+\hat{b}_2\cdot 0)$$

or

$$= \frac{1}{1+\exp(-\hat{a}-\hat{b}_1X_1-\hat{b}_2\cdot 1)} - \frac{1}{1+\exp(-\hat{a}-\hat{b}_1X_1-\hat{b}_2\cdot 0)}$$

- Actually, it is safe to use the prediction method to evaluate partial (marginal) effects in all cases.

$$\frac{dP}{dX_1} = F(\hat{a}+\hat{b}_1(X_1+1)+\hat{b}_2D) - F(\hat{a}+\hat{b}_1X_1+\hat{b}_2D)$$

$$\simeq F(\cdot)\hat{b}_1 \quad \text{approximately the same}$$