# Regression with a Binary Dependent Variable
# (SW Ch. 9)

**EC 471**
**Spring 2004**



## Regression with a Binary Dependent Variable (SW Ch. 9)

So far the dependent variable ($Y$) has been continuous:
- district-wide average test score
- traffic fatality rate

But we might want to understand the effect of $X$ on a binary variable:
- $Y$ = get into college, or not
- $Y$ = person smokes, or not
- $Y$ = mortgage application is accepted, or not



## Example: Mortgage denial and race

**The Boston Fed HMDA data set**
- Individual applications for single-family mortgages made in 1990 in the greater Boston area
- 2380 observations, collected under Home Mortgage Disclosure Act (HMDA)

**Variables**
- Dependent variable:
  - Is the mortgage denied or accepted?
- Independent variables:
  - income, wealth, employment status
  - other loan, property characteristics
  - race of applicant



## The Linear Probability Model
## (SW Section 9.1)

A natural starting point is the linear regression model with a single regressor:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

But:
- What does the predicted value $\hat{Y}$ mean when $Y$ is binary? For example, what does $\hat{Y} = 0.26$ mean?
- What does $\beta_1$ mean when $Y$ is binary? Is $\beta_1 = \frac{\Delta Y}{\Delta X}$?

## The linear probability model, ctd.

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

Recall assumption #1: $E(u_i|X_i) = 0$, so

$$E(Y_i|X_i) = E(\beta_0 + \beta_1 X_i + u_i|X_i) = \beta_0 + \beta_1 X_i$$

When $Y$ is binary,

$$E(Y) = 1\times\Pr(Y=1) + 0\times\Pr(Y=0) = \Pr(Y=1)$$

so

$$E(Y|X) = \Pr(Y=1|X)$$



## The linear probability model, ctd.

When $Y$ is binary, the linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

is called the ***linear probability model***.

- The predicted value is a ***probability***:
  - $E(Y|X=x) = \Pr(Y=1|X=x)$ = prob. that $Y = 1$ given $x$
  - $\hat{Y}$ = the ***predicted probability*** that $Y_i = 1$, given $X$
- $\beta_1$ = change in probability that $Y = 1$ for a given $\Delta x$:

$$\beta_1 = \frac{\Pr(Y=1|X=x+\Delta x) - \Pr(Y=1|X=x)}{\Delta x}$$

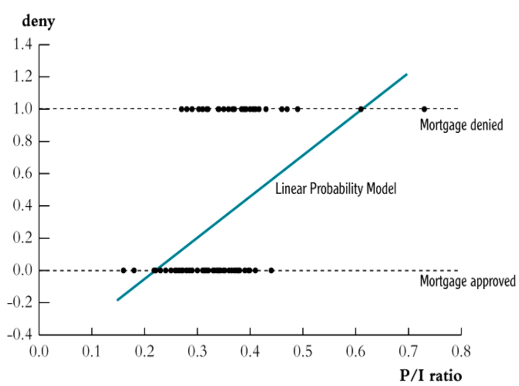
Example: linear probability model, HMDA data



## Mortgage denial v. ratio of debt payments to income (P/I ratio) in the HMDA data set (subset)

**FIGURE 9.1** Scatterplot of Mortgage Application Denial and the Payment-to-Income Ratio

Mortgage applicants with a high ratio of debt payments to income (*P/I ratio*) are more likely to have their application denied (*deny* = 1 if denied, *deny* = 0 if approved). The linear probability model uses a straight line to model the probability of denial, conditional on the *P/I ratio*.






## Linear probability model: HMDA data

$$\widehat{deny} = -.080 + .604\ P/I\ ratio \qquad (n = 2380)$$
$$(.032)\ (.098)$$

- What is the predicted value for *P/I ratio* = .3?

  $\widehat{\Pr(deny=1|P/Iratio=.3)}$ = -.080 + .604×.3 = .151

- Calculating "effects:" increase *P/I ratio* from .3 to .4:

  $\widehat{\Pr(deny=1|P/Iratio=.4)}$ = -.080 + .604×.4 = .212

  The effect on the probability of denial of an increase in *P/I ratio* from .3 to .4 is to increase the probability by .061, that is, by 6.1 *percentage points* (*what?*).

Next include *black* as a regressor:

$$\widehat{deny} = -.091 + .559 P/I\ ratio + .177 black$$
$$(.032)\quad (.098)\qquad\qquad (.025)$$

Predicted probability of denial:

- for black applicant with *P/I ratio* = .3:

$$\widehat{\Pr(deny=1)} = -.091 + .559\times.3 + .177\times1 = .254$$

- for white applicant, *P/I ratio* = .3:

$$\widehat{\Pr(deny=1)} = -.091 + .559\times.3 + .177\times0 = .077$$

- **difference** = .177 = 17.7 percentage points
- Coefficient on *black* is significant at the 5% level
- *Still plenty of room for omitted variable bias…*



**The linear probability model: Summary**

- Models probability as a linear function of $X$
- Advantages:
  - simple to estimate and to interpret
  - inference is the same as for multiple regression (*need heteroskedasticity-robust standard errors*)
- Disadvantages:
  - Does it make sense that the probability should be linear in X?
  - Predicted probabilities can be <0 or >1!
- These disadvantages can be solved by using a nonlinear probability model: probit and logit regression



## Probit and Logit Regression (SW Section 9.2)

The problem with the linear probability model is that it models the probability of $Y$=1 as being linear:

$$\Pr(Y = 1|X) = \beta_0 + \beta_1 X$$

Instead, we want:

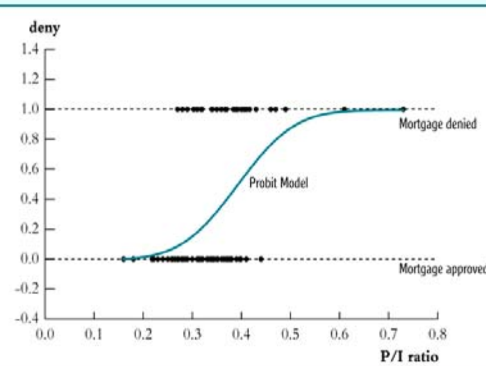
- $0 \le \Pr(Y = 1|X) \le 1$ for all $X$
- $\Pr(Y = 1|X)$ to be increasing in $X$ (for $\beta_1>0$)

This requires a *nonlinear* functional form for the probability. How about an "S-curve"…



**FIGURE 9.2** Probit Model of the Probability of Denial, Given the P/I Ratio

The probit model uses the cumulative normal distribution function to model the probability of denial given the payment-to-income ratio or, more generally, to model Pr(Y = 1 | X). Unlike the linear probability model, the probit conditional probabilities are always between zero and one.




The probit model satisfies these conditions:

- $0 \le \Pr(Y = 1|X) \le 1$ for all $X$
- $\Pr(Y = 1|X)$ to be increasing in $X$ (for $\beta_1>0$)

***Probit regression*** models the probability that $Y$=1 using the cumulative standard normal distribution function, evaluated at $z = \beta_0 + \beta_1 X$:

$$\Pr(Y = 1|X) = \Phi(\beta_0 + \beta_1 X)$$

- $\Phi$ is the **cumulative normal distribution function**.
- $z = \beta_0 + \beta_1 X$ is the "$z$-value" or "$z$-index" of the probit model.

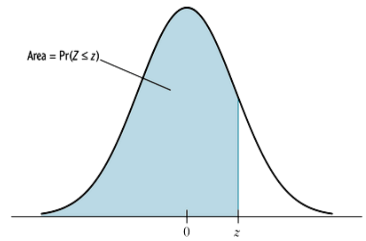
*Example*: Suppose $\beta_0 = -2$, $\beta_1 = 3$, $X = .4$, so

$$\Pr(Y = 1|X=.4) = \Phi(-2 + 3 \times .4) = \Phi(-0.8)$$

$\Pr(Y = 1|X=.4)$ = area under the standard normal density to left of $z$ = -.8, which is…



**TABLE 1** The Cumulative Standard Normal Distribution Function, $\Phi(z) = \Pr(Z \le z)$




| $z$ | Second Decimal Value of $z$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |

$$\Pr(Z \le -0.8) = .2119$$



**Probit regression, ctd.**

Why use the cumulative normal probability distribution?

- The "S-shape" gives us what we want:
  - $0 \le \Pr(Y = 1|X) \le 1$ for all $X$
  - $\Pr(Y = 1|X)$ to be increasing in $X$ (for $\beta_1 > 0$)
- Easy to use – the probabilities are tabulated in the cumulative normal tables
- Relatively straightforward interpretation:
  - $z$-value = $\beta_0 + \beta_1 X$
  - $\hat{\beta}_0 + \hat{\beta}_1 X$ is the predicted $z$-value, given $X$
  - $\beta_1$ is the change in the $z$-value for a unit change in $X$



*STATA Example*: HMDA data

```
. probit deny p_irat, r;

Iteration 0:   log likelihood =  -872.0853      We'll discuss this later
Iteration 1:   log likelihood =  -835.6633
Iteration 2:   log likelihood = -831.80534
Iteration 3:   log likelihood = -831.79234

Probit estimates                                  Number of obs   =       2380
                                                  Wald chi2(1)    =      40.68
                                                  Prob > chi2     =     0.0000
Log likelihood = -831.79234                       Pseudo R2       =     0.0462

------------------------------------------------------------------------------
             |               Robust
        deny |      Coef.   Std. Err.      z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_irat |   2.967908   .4653114     6.38   0.000      2.055914    3.879901
       _cons |  -2.194159   .1649721   -13.30   0.000     -2.517499    -1.87082
------------------------------------------------------------------------------
```

$$\widehat{\Pr(deny=1|P/Iratio)} = \Phi(-2.19 + 2.97 \times P/I\ ratio)$$

$$(.16) \quad (.47)$$

## *STATA Example*: HMDA data, ctd.

$$\widehat{\Pr(deny=1|P/Iratio)} = \Phi(-2.19 + 2.97\times P/I\ ratio)$$

$$(.16) \quad (.47)$$

- Positive coefficient: *does this make sense*?
- Standard errors have usual interpretation
- Predicted probabilities:

$$\widehat{\Pr(deny=1|P/Iratio=.3)} = \Phi(-2.19+2.97\times .3)$$

$$= \Phi(-1.30) = .097$$

- Effect of change in *P/I ratio* from .3 to .4:

$$\widehat{\Pr(deny=1|P/Iratio=.4)} = \Phi(-2.19+2.97\times .4) = .159$$

Predicted probability of denial rises from .097 to .159



## Probit regression with multiple regressors

$$\Pr(Y = 1|X_1, X_2) = \Phi(\beta_0 + \beta_1 X_1 + \beta_2 X_2)$$

- $\Phi$ is the cumulative normal distribution function.
- $z = \beta_0 + \beta_1 X_1 + \beta_2 X_2$ is the "$z$-value" or "$z$-index" of the probit model.
- $\beta_1$ is the effect on the $z$-score of a unit change in $X_1$, holding constant $X_2$



## *STATA Example*: HMDA data

```
. probit deny p_irat black, r;

Iteration 0:   log likelihood =  -872.0853
Iteration 1:   log likelihood = -800.88504
Iteration 2:   log likelihood =  -797.1478
Iteration 3:   log likelihood = -797.13604

Probit estimates                                  Number of obs   =       2380
                                                  Wald chi2(2)    =     118.18
                                                  Prob > chi2     =     0.0000
Log likelihood = -797.13604                       Pseudo R2       =     0.0859

------------------------------------------------------------------------------
             |               Robust
        deny |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_irat |   2.741637   .4441633     6.17   0.000     1.871092    3.612181
       black |   .7081579   .0831877     8.51   0.000      .545113    .8712028
       _cons |  -2.258738   .1588168   -14.22   0.000    -2.570013   -1.947463
------------------------------------------------------------------------------
```

*We'll go through the estimation details later…*



## *STATA Example*: predicted probit probabilities

```
. probit deny p_irat black, r;

Probit estimates                                  Number of obs   =       2380
                                                  Wald chi2(2)    =     118.18
                                                  Prob > chi2     =     0.0000
Log likelihood = -797.13604                       Pseudo R2       =     0.0859

------------------------------------------------------------------------------
             |               Robust
        deny |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_irat |   2.741637   .4441633     6.17   0.000     1.871092    3.612181
       black |   .7081579   .0831877     8.51   0.000      .545113    .8712028
       _cons |  -2.258738   .1588168   -14.22   0.000    -2.570013   -1.947463
------------------------------------------------------------------------------

.  sca z1 = _b[_cons]+_b[p_irat]*.3+_b[black]*0;

.  display "Pred prob, p_irat=.3, white: "  normprob(z1);

Pred prob, p_irat=.3, white: .07546603

       NOTE
         _b[_cons] is the estimated intercept (-2.258738)
         _b[p_irat] is the coefficient on p_irat (2.741637)
         sca creates a new scalar which is the result of a calculation
         display prints the indicated information to the screen
```

*STATA Example*: HMDA data, ctd.

$\widehat{\Pr(deny=1|P/I, black)}$

$= \Phi(-2.26 + 2.74 \times P/I\ ratio + .71 \times black)$

(.16) (.44) (.08)

- Is the coefficient on *black* statistically significant?
- Estimated effect of race for *P/I ratio* = .3:

$\widehat{\Pr(deny=1|.3,1)} = \Phi(-2.26+2.74\times.3+.71\times1) = .233$

$\widehat{\Pr(deny=1|.3,0)} = \Phi(-2.26+2.74\times.3+.71\times0) = .075$

- Difference in rejection probabilities = .158 (15.8 percentage points)
- *Still plenty of room still for omitted variable bias…*



# Logit regression

***Logit regression*** models the probability of $Y$=1 as the cumulative standard *logistic* distribution function, evaluated at $z = \beta_0 + \beta_1 X$:

$$\Pr(Y = 1|X) = F(\beta_0 + \beta_1 X)$$

$F$ is the **cumulative logistic distribution function**:

$$F(\beta_0 + \beta_1 X) = \frac{1}{1+e^{-(\beta_0+\beta_1 X)}}$$



**Logistic regression, ctd.**

$$\Pr(Y = 1|X) = F(\beta_0 + \beta_1 X)$$

where $F(\beta_0 + \beta_1 X) = \frac{1}{1+e^{-(\beta_0+\beta_1 X)}}$.

*Example*: $\beta_0 = -3$, $\beta_1 = 2$, $X = .4$,

so $\beta_0 + \beta_1 X = -3 + 2\times.4 = -2.2$

$\Pr(Y = 1|X=.4) = \mathbf{1/(1+}e^{-(-2.2)}\mathbf{)} = .0998$

Why bother with logit if we have probit?

- Historically, numerically convenient
- In practice, very similar to probit



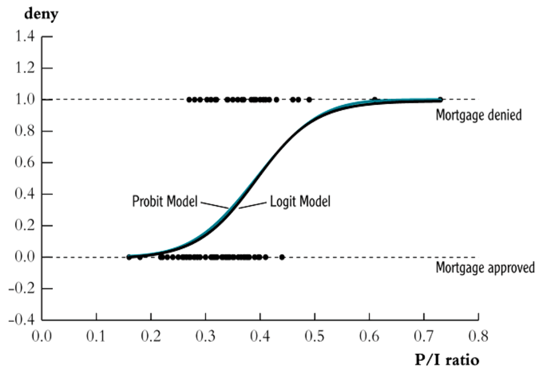
*STATA Example*: HMDA data

```
. logit deny p_irat black, r;

Iteration 0:   log likelihood =  -872.0853      Later…
Iteration 1:   log likelihood =  -806.3571
Iteration 2:   log likelihood = -795.74477
Iteration 3:   log likelihood = -795.69521
Iteration 4:   log likelihood = -795.69521

Logit estimates                                   Number of obs   =       2380
                                                  Wald chi2(2)    =     117.75
                                                  Prob > chi2     =     0.0000
Log likelihood = -795.69521                       Pseudo R2       =     0.0876

------------------------------------------------------------------------------
             |               Robust
        deny |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      p_irat |   5.370362   .9633435     5.57   0.000     3.482244    7.258481
       black |   1.272782   .1460986     8.71   0.000     .9864339     1.55913
       _cons |  -4.125558    .345825   -11.93   0.000    -4.803362   -3.447753
------------------------------------------------------------------------------

.  dis "Pred prob, p_irat=.3, white: "
>     1/(1+exp(-(_b[_cons]+_b[p_irat]*.3+_b[black]*0)));

Pred prob, p_irat=.3, white: .07485143
         NOTE:  the probit predicted probability is .07546603
```

Predicted probabilities from estimated probit and logit models usually are very close.

**FIGURE 9.3** Probit and Logit Models of the Probability of Denial, Given the P/I Ratio

These logit and probit models produce nearly identical estimates of the probability that a mortgage application will be denied, given the payment-to-income ratio.






**Estimation and Inference in Probit (and Logit) Models (SW Section 9.3)**

Probit model:

$$\Pr(Y = 1|X) = \Phi(\beta_0 + \beta_1 X)$$

- Estimation and inference
  - How to estimate $\beta_0$ and $\beta_1$?
  - What is the sampling distribution of the estimators?
  - Why can we use the usual methods of inference?
- First discuss *nonlinear least squares* (easier to explain)
- Then discuss *maximum likelihood* estimation (what is actually done in practice)



**Probit estimation by nonlinear least squares**

Recall OLS:

$$\min_{b_0,b_1} \sum_{i=1}^{n} [Y_i - (b_0 + b_1 X_i)]^2$$

- The result is the OLS estimators $\hat{\beta}_0$ and $\hat{\beta}_1$

In probit, we have a different regression function – the nonlinear probit model. So, we could estimate $\beta_0$ and $\beta_1$ by ***nonlinear least squares***:

$$\min_{b_0,b_1} \sum_{i=1}^{n} [Y_i - \Phi(b_0 + b_1 X_i)]^2$$

Solving this yields the ***nonlinear least squares*** estimator of the probit coefficients.



**Nonlinear least squares, ctd.**

$$\min_{b_0,b_1} \sum_{i=1}^{n} [Y_i - \Phi(b_0 + b_1 X_i)]^2$$

How to solve this minimization problem?

- Calculus doesn't give and explicit solution.
- Must be solved *numerically* using the computer, e.g. by "trial and error" method of trying one set of values for ($b_0$,$b_1$), then trying another, and another,…
- Better idea: use specialized minimization algorithms

In practice, nonlinear least squares isn't used because it isn't efficient – an estimator with a smaller variance is…

## Probit estimation by maximum likelihood

The ***likelihood function*** is the conditional density of $Y_1,\ldots,Y_n$ given $X_1,\ldots,X_n$, treated as a function of the unknown parameters $\beta_0$ and $\beta_1$.

- The maximum likelihood estimator (MLE) is the value of ($\beta_0$, $\beta_1$) that maximize the likelihood function.
- The MLE is the value of ($\beta_0$, $\beta_1$) that best describe the full distribution of the data.
- In large samples, the MLE is:
  - consistent
  - normally distributed
  - efficient (has the smallest variance of all estimators)



## Special case: the probit MLE with no *X*

$$Y = \begin{cases} 1 \text{ with probability } p \\ 0 \text{ with probability } 1-p \end{cases} \quad \text{(Bernoulli distribution)}$$

Data: $Y_1,\ldots,Y_n$, i.i.d.

Derivation of the likelihood starts with the density of $Y_1$:

$$\Pr(Y_1 = 1) = p \text{ and } \Pr(Y_1 = 0) = 1-p$$

so

$$\Pr(Y_1 = y_1) = p^{y_1}(1-p)^{1-y_1} \quad \textit{(verify this for } y_1\textit{=0, 1!)}$$



Joint density of $(Y_1,Y_2)$:
Because $Y_1$ and $Y_2$ are independent,

$$\Pr(Y_1 = y_1, Y_2 = y_2) = \Pr(Y_1 = y_1) \times \Pr(Y_2 = y_2)$$
$$= [p^{y_1}(1-p)^{1-y_1}] \times [p^{y_2}(1-p)^{1-y_2}]$$

Joint density of $(Y_1,..,Y_n)$:

$$\Pr(Y_1 = y_1, Y_2 = y_2, \ldots, Y_n = y_n)$$
$$= [p^{y_1}(1-p)^{1-y_1}] \times [p^{y_2}(1-p)^{1-y_2}] \times \ldots \times [p^{y_n}(1-p)^{1-y_n}]$$
$$= p^{\sum_{i=1}^n y_i}(1-p)^{\left(n-\sum_{i=1}^n y_i\right)}$$



The likelihood is the joint density, treated as a function of the unknown parameters, which here is $p$:

$$f(p;Y_1,\ldots,Y_n) = p^{\sum_{i=1}^n Y_i}(1-p)^{\left(n-\sum_{i=1}^n Y_i\right)}$$

The MLE maximizes the likelihood. Its standard to work with **the log likelihood**, $\ln[f(p;Y_1,\ldots,Y_n)]$:

$$\mathbf{\ln[f(p;Y_1,\ldots,Y_n)]} = \left(\sum_{i=1}^n Y_i\right)\ln(p) + \left(n - \sum_{i=1}^n Y_i\right)\ln(1-p)$$

**The probit likelihood with one $X$**

The derivation starts with the density of $Y_1$, given $X_1$:

$$\Pr(Y_1 = 1|X_1) = \Phi(\beta_0 + \beta_1 X_1)$$

$$\Pr(Y_1 = 0|X_1) = 1-\Phi(\beta_0 + \beta_1 X_1)$$

so

$$\Pr(Y_1 = y_1|X_1) = \Phi(\beta_0+\beta_1 X_1)^{y_1}[1-\Phi(\beta_0+\beta_1 X_1)]^{1-y_1}$$

The probit likelihood function is the joint density of $Y_1,\ldots,Y_n$ given $X_1,\ldots,X_n$, treated as a function of $\beta_0$, $\beta_1$:

$$f(\beta_0,\beta_1;\, Y_1,\ldots,Y_n|X_1,\ldots,X_n)$$

$$= \{\Phi(\beta_0+\beta_1 X_1)^{Y_1}[1-\Phi(\beta_0+\beta_1 X_1)]^{1-Y_1}\}\times$$

$$\ldots\times\{\Phi(\beta_0+\beta_1 X_n)^{Y_n}[1-\Phi(\beta_0+\beta_1 X_n)]^{1-Y_n}\}$$



**The probit likelihood function**:

$$f(\beta_0,\beta_1;\, Y_1,\ldots,Y_n|X_1,\ldots,X_n)$$

$$= \{\Phi(\beta_0+\beta_1 X_1)^{Y_1}[1-\Phi(\beta_0+\beta_1 X_1)]^{1-Y_1}\}\times$$

$$\ldots\times\{\Phi(\beta_0+\beta_1 X_n)^{Y_n}[1-\Phi(\beta_0+\beta_1 X_n)]^{1-Y_n}\}$$

- Can't solve for the maximum explicitly
- Must maximize using numerical methods
- As in the case of no $X$, in large samples:
  - $\hat{\beta}_0^{MLE}$, $\hat{\beta}_1^{MLE}$ are consistent
  - $\hat{\beta}_0^{MLE}$, $\hat{\beta}_1^{MLE}$ are normally distributed (more later…)
  - Their standard errors can be computed
  - Testing, confidence intervals proceeds as usual
- For multiple $X$'s, see SW App. 9.2



**Measures of fit**

The $R^2$ and $\bar{R}^2$ don't make sense here (*why?*). So, two other specialized measures are used:

1. The ***fraction correctly predicted*** = fraction of $Y$'s for which predicted probability is >50% (if $Y_i$=1) or is <50% (if $Y_i$=0).

2. The ***pseudo-$R^2$*** measure the fit using the likelihood function: measures the improvement in the value of the log likelihood, relative to having no $X$'s (see SW App. 9.2). This simplifies to the $R^2$ in the linear model with normally distributed errors.

Note (Lee): eople report these values, but practically, they carry no important meaning, I presume.



**Testing Hypothesis**

**(a) One restriction on each coefficient**

Use usual t-stat.

**(b) More than one restriction**

Use Likelihood Ratio (LR) Test

.. Chi-square ($\chi$) test..

**LR = 2 ($\log L_u - \log L_R$)** $\sim$ $\chi$ with df = g

$\log L_u$ = Unrestricted log-lik

$\log L_R$ = Restricted log-lik

# Example: Extra-marital affairs

## Y = 1 (affairs), or 0 (no)

```
. log using "C:\EC471\fair.log"

. insheet using "C:\EC471\fair.txt"
(13 vars, 601 obs)

. probit y  age edu kids occupation rating_m religion sex yrs_marr

Iteration 0:   log likelihood = -337.68849
Iteration 1:   log likelihood = -305.53338
Iteration 2:   log likelihood = -305.19816
Iteration 3:   log likelihood = -305.19796

Probit estimates                                  Number of obs   =        601
                                                  LR chi2(8)      =      64.98
                                                  Prob > chi2     =     0.0000
Log likelihood = -305.19796                       Pseudo R2       =     0.0962

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |  -.0245844   .0104178    -2.36   0.018    -.0450028   -.0041659
         edu |   .0112622   .0295165     0.38   0.703    -.0465891    .0691135
```



```
        kids |   .2166441   .1651681     1.31   0.190    -.1070795    .5403677
  occupation |   .0136686   .0414037     0.33   0.741    -.0674813    .0948184
    rating_m |  -.2717912   .0534747    -5.08   0.000    -.3765998   -.1669826
    religion |  -.1854684   .0516258    -3.59   0.000    -.2866532   -.0842837
         sex |   .1734569   .1379911     1.26   0.209    -.0970007    .4439144
    yrs_marr |   .0543435   .0188087     2.89   0.004     .0174792    .0912078
       _cons |   .7794021   .5125492     1.52   0.128    -.2251759     1.78398
------------------------------------------------------------------------------
```

**Now, we wish to test if the coefficients of AGE and EDU are jointly insignificant.**

**$H_o$: $\beta_1 = \beta_2 = 0$ $H_a$: $H_o$ is not true**

**Unrestricted Log likelihood = -305.19796**

**We run a restricted probit model (without age and edu).**

```
. probit y   kids occupation rating_m religion sex yrs_marr

Iteration 0:   log likelihood = -337.68849
Iteration 1:   log likelihood = -308.34709
Iteration 2:   log likelihood = -308.08918
Iteration 3:   log likelihood = -308.08906

Probit estimates                                  Number of obs   =        601
```



```
                                                  LR chi2(6)      =      59.20
                                                  Prob > chi2     =     0.0000
Log likelihood = -308.08906                       Pseudo R2       =     0.0877

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        kids |   .2367733   .1639708     1.44   0.149    -.0846035    .5581501
  occupation |   .0134218   .0369199     0.36   0.716    -.0589399    .0857835
    rating_m |  -.2642769   .0522959    -5.05   0.000     -.366775   -.1617789
    religion |   -.188488   .0514507    -3.66   0.000    -.2893295   -.0876464
         sex |   .1174134   .1325305     0.89   0.376    -.1423415    .3771684
    yrs_marr |   .0223682   .0130136     1.72   0.086     -.003138    .0478744
       _cons |   .4233281   .3149532     1.34   0.179    -.1939689    1.040625
------------------------------------------------------------------------------
```

**Restricted Log likelihood = −308.08906**

**LR = 2(log$L_U$ − log$L_R$)= 2 * (-305.19796 − (−308.08906))**
**= 5.65**

**5% Critical value of the chi-square test with df 2 = 5.99**

**Thus, we do not reject the null. The two coefficients are jointly insignificant.**



## Simply, in STATA

```
. test age edu

 ( 1)  age = 0
 ( 2)  edu = 0

           chi2(  2) =    5.65
         Prob > chi2 =    0.0594
```

## Note: overall insignificance (all slope coefficients = 0)

```
LR chi2(8)      =      64.98    Prob > chi2     =     0.0000
```

**Summary**
**(SW Section 9.5)**

- If $Y_i$ is binary, then $E(Y|X) = \Pr(Y=1|X)$
- Three models:
  - linear probability model (linear multiple regression)
  - probit (cumulative standard normal distribution)
  - logit (cumulative standard logistic distribution)
- LPM, probit, logit all produce predicted probabilities
- Effect of $\Delta X$ is change in conditional probability that $Y=1$. For logit and probit, this depends on the initial $X$
- Probit and logit are estimated via maximum likelihood
  - Coefficients are normally distributed for large $n$
  - Large-$n$ hypothesis testing, conf. intervals is as usual