

**UNIVERSITÄT KONSTANZ****Insurance Management**

Oktober 2007

Professor Schlesinger

**KLAUSUR**

Answer **any 5** of the following 6 questions. Answers may be written in German or in English. Each question answered is worth 20 points (100 points in total). To get full credit you must show how you derive your answers. You need to earn at least 50 points to pass the course. Note that I will not grade all 6 questions, so only answer 5. **Please write neatly.**

Use the following abbreviated mortality table to answer question 1.

Age	Probability of Dying	Number of People	Number of Deaths
40	0.00302	937,723	2832
41	0.00329	934,891	3076
42	0.00356	931,815	3317
43	0.00387	928,498	3593
44	0.00419	924,905	3875

1. Jan just turned 41 years old and wishes to purchase a three-year decreasing term life insurance policy. This policy pays € 100,000 if death occurs during the first year of the policy; € 90,000 if death occurs during the second year; and € 80,000 if death occurs during the third year. Any premium is paid at the beginning of the year, and death benefits are paid out at the end of the year of death. Assume that the interest rate for borrowing and lending is 10%. Further assume that there are no premium loading charges.

- Determine the single premium that he would be charged for such a policy.
- Determine the level premium that he would be charged for such a policy.

2. A risk-averse art collector owns two original Picasso paintings (painted by Diego Picasso, no relation to Pablo). Each painting is worth € 40 and each has a probability  $p = \frac{1}{4}$  of being stolen. Because the paintings are kept at different locations, these risks are independent from one another. Deductible insurance is available at a fair price, but the level of insurance is limited to a total insurance premium of € 10.

- Suppose that the collector must purchase a separate policy for each painting, with deductible levels  $D_1$  and  $D_2$  respectively. Show that spending an equal insurance premium of € 5 per painting is optimal.
- Suppose that the collector is allowed to use her € 10 to purchase one policy for both paintings, with an aggregate deductible  $D$ . Determine the level of  $D$  and show whether or not this aggregate deductible is preferred to the two separate deductibles.
- If this consumer could have any size insurance budget, not just a budget of € 10, what would that budget be? Which would be better in this case, two separate deductibles or one aggregate deductible? Explain.

**YOU MUST TURN IN THIS QUESTION SHEET WITH YOUR EXAM!**

3. A risk averse consumer has expected-utility preferences that exhibit constant absolute risk aversion (CARA). The consumer currently purchases coinsurance for her private airplane, with an optimal coinsurance rate of  $\alpha^* = 0.75$ .

(a) Suppose that this consumer's wealth is subject to a non-insurable independent zero-mean background risk  $\tilde{\varepsilon}$ . Will the optimal coinsurance level for her airplane change? If so, will it be more or less than  $\alpha = 0.75$ ? You must show how you derive your answer.

(b) Suppose that this consumer's wealth is subject to a non-insurable independent background risk  $\tilde{\varepsilon}$ , but with a positive mean  $E\tilde{\varepsilon} > 0$ . Will the optimal coinsurance level for her airplane change? If so, will it be more or less than  $\alpha = 0.75$ ? You must show how you derive your answer.

4. An individual has a wealth of 100 that is subjected to a random loss. The loss has a uniform density on  $[0,100]$ ,  $f(x) = 0.01$  for all  $x \in [0,100]$  and  $f(x) = 0$  elsewhere.

(a) Assume no premium loading,  $\lambda = 0$ . Compute the premium corresponding to a deductible  $D = 40$ .

(b) Consider now a policy with an upper limit  $\theta$  so that:

$$I(x) = x \text{ if } x \leq \theta \\ = \theta \text{ if } x \geq \theta$$

Which value of  $\theta$  yields the same premium as in part a?

(c) By drawing cumulative distributions of final wealth show that a risk averter will prefer the deductible to the upper limit. Explain your answer carefully.

5. Mr. Kuhfladen is risk-averse with initial wealth  $W$ . He faces a loss of size  $L$  with probability  $p$ , where  $0 < p < 1$  and  $0 < L < W$ . Assume that coinsurance is available at any coinsurance rate  $\alpha$ ,  $0 \leq \alpha \leq 1$ . The premium for this coverage is  $P(\alpha) = (1 + \lambda)\alpha pL$ , with  $\lambda > 0$ . Mr. Kuhfladen's preferences can be represented by the so-called "dual" preference functional

$$V(F) = \int y d[g(F(y))],$$

where  $F(y)$  is the cumulative distribution function of final wealth  $y$ , and  $g$  is an increasing and strictly convex function such that  $g(0) = 0$  and  $g(1) = 1$ .

Determine the optimal level of coinsurance. In particular, you should determine whether zero insurance, partial insurance or full insurance will be optimal.

6. Consider a two-state model of insurance demand. The insured either suffers a loss of size  $L$ , or else there is no loss at all. The insured can either take effort or not take effort. If the insured takes effort, the “cost” is  $c$  units of utility. The benefit of taking effort is that it lowers the probability of a loss from  $p_N$  to  $p_E$ , where  $0 < p_E < p_N < 1$ . The insurer cannot observe whether or not effort was taken, but knows the individual’s incentives. (Note: assume that the consumer takes effort whenever she is indifferent between effort and no effort.)

(a) Suppose the insurer charges a fair premium for any level of insurance, where the fair premium is based on the individual’s incentive to take effort or not take effort. Carefully describe the possible equilibrium levels of insurance. Explain your reasoning.

(b) Suppose the cost of effort increases to  $d$  units of utility, where  $d > c$ . Will the level of insurance purchased rise, fall or stay the same? Explain carefully.

*PLEASE PUT YOUR MATRIKEL-NUMBER ON THE FIRST PAGE*

*PLEASE CHECK TO BE SURE THAT YOU ANSWERED 5 (AND ONLY 5) QUESTIONS*

**YOU MUST TURN IN THIS QUESTION SHEET WITH YOUR EXAM!**