Insurance Management

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Problems:

1. Consider distribution functions with supports contained in (a,b). Define the following preference functionals:

(i) $V(F) \equiv \mu_F$ (the mean of F)

(ii) $V(F) \equiv$ median of F

(iii) $V(F) \equiv$ max Supp (F)

(iv) $V(F) \equiv$ min Supp (F)

(v) $V(F) \equiv f(\mu_F, \sigma_F^2))$ (function of mean and variance)

For preference functionals (i)-(iv) above, find examples of simple lotteries to convince yourself that these preferences are unrealistic. Do you think preference functional (v) is realistic? Why or why not?

2. Lottery A yields either 0 or 10 units of utility, each with an equal probability. Lottery B yields 5 units of utility and no risk. Which lottery is preferred by a risk averter and why?

3. Assume $u''' > 0$ and define absolute prudence as $p(y) \equiv \frac{-u'''(y)}{u''(y)}$.

Define $v(y) \equiv -u'(y)$ and note that $v(y)$ has the properties of a risk-averse utility function. Show the equivalence of:

(a) u exhibits DARA
(b) $p(y) > r(y) \forall y$
(c) $v(y)$ is more risk averse than $u(y)$.

4. A consumer has \$100 in wealth plus a lottery ticket. The lottery ticket pays a prize of \$100 with probability p = ½. Otherwise, the prize is zero. The consumer has von Neumann-Morgenstern utility exhibiting constant relative risk aversion, with the level of relative risk aversion γ=1.

(a) What is the lowest price that consumer would accept to sell this lottery ticket?

(b) If the consumer had \$100, but did not own the lottery ticket, how much would she be willing to pay to buy the ticket?

5. Suppose that $F \sim Unif[a,b]$ and that $G \sim Unif[A,B]$. Give conditions on {a, b, A, B} such that F FSD G. Give conditions such that F SSD G.

6. Let $\tilde{X}$ denote an equally weighted lottery with prizes 5 and 10. . Let $\tilde{Y}$ denote an equally weighted lottery with prizes 0 and 15. Explain how $\tilde{Y}$ is a simple mean-preserving spread of $\tilde{X}$. Find $\tilde{\varepsilon}$ such that $E(\tilde{\varepsilon} \mid x) = 0 \ \forall x$ and $\tilde{Y} \stackrel{d}{=} \tilde{X} + \tilde{\varepsilon}$.

7. Both $\tilde{x}$ and $\tilde{y}$ have supports contained in (a,b). Show that $E\tilde{x} = E\tilde{y}$ if and only if $\int_a^b F(x)dx = \int_a^b G(x)dx$.

8. Consider a two-state loss model, where a loss of size L occurs with probability p, $L \le W$. Let $v(y)$ be a more risk-averse utility than $u(y)$. Show the following:

(a) If the premium loading $\lambda = 0$, then $\alpha_u^* = \alpha_v^* = 1$.
(b) If $\lambda > 0$, then $\alpha_u^* \le \alpha_v^* < 1$.
(Hint: Use Pratt's Theorem.)

9. Let $\tilde{Y} = W - \tilde{x}$, with insurance available for the loss $\tilde{x}$ and with $\lambda > 0$. Let α^* denote the optimal coinsurance level.

(a) Suppose that preferences satisfy CARA. Show that α^* remains unchanged for $\tilde{Y} = kw - \tilde{x}$, $k>0$.
(b) Suppose that preferences satisfy CRRA. Show that α^* remains unchanged for $\tilde{Y} = k\tilde{w} - k\tilde{x}$, $k > 0$.

10. Consider a state-claims model with two states of nature. What is the slope of the price line for insurance contracts? Use this information to demonstrate Mossin's Theorem.

11. Let $u(y) \equiv y - ky^2, k > 0, y < 1/2k$. Let $\tilde{\varepsilon}$ be an independent background risk, $E\tilde{\varepsilon} = 0$. Show that the optimal level of insurance α^* is the same both with and without the background risk. (Note that u''' is zero for quadratic utility.)

12. Consider a model of insurance demand with a ''fair'' price and a possibility of insurer default. Show that an increase in risk aversion might <u>not</u> lead to an increase in the optimal level of insurance.

13. Consider a model of deductible insurance. Show that Mossin's Theorem also holds for deductibles:
(1) $\lambda = 0 \Rightarrow$ Full insurance is optimal, $D^* = 0$.
(2) $\lambda > 0 \Rightarrow$ Partial coverage is optimal, $D^* > 0$.

19. Consider the Rothschild-Stiglitz adverse-selection model, but with 3 types of insureds: good, medium, and bad, where $p_G < p_M < p_B$. Characterize the R-S equilibrium. Be sure to consider pooling contracts, separating contracts and mixed contracts (i.e. where two types pool with a separate contract for the third type.)

20. Consider a two-state model of insurance demand. The insured has two possible levels of effort. Suppose insurance is offered, but at a price that includes a premium loading $\lambda > 0$. The loading λ can be thought of as a competitive loading to cover marketing expenses. Characterize insurance prices and insurance demand under moral hazard.

21. Should European insurance-company regulations be imposed by the countries in which the insurers are domiciled? Or should there be a uniform regulation throughout the EU? Discuss the advantages and disadvantages of each approach.