EC 623 Fall 2009 Gary A. Hoover

Problem Set III

1. For small discrete populations, the T measure may be written as:

$$T = \ln\left(\sum_{i=1}^{n} S_i Y_i\right) - \sum_{i=1}^{n} S_i \ln(Y_i),$$

Where Y_i is the income of the ith income recipient in a population of n income recipients. S_i is the share of Y_i in total income of all income recipients. That is:

$$S_i = Y_i / \sum_{j=1}^{n} Y_j$$

Suppose an island population consists of three individuals with incomes: Y_1 = \$95, Y_2 = \$40 and Y_3 = \$15.

a) Compute the T measure of inequality.
b) Increase each of the incomes by 20% and recompute the T measure.
c) Suppose Y_1 = \$85, Y_2 = \$40 and Y_3 = \$25, because the richest person gave \$10 to the poorest person. Recompute the T measure
d) Suppose Y_1 = \$50, Y_2 = \$50 and Y_3 = \$50. Recompute the T measure.
e) Comment on your results.

2. Suppose that an individual has utility function $U = AC^{a}L^{1-a}$. He has non-wage income \$$V$, and a total of T hours each week to allocate between work, h, and leisure, L. He can work as many hours as he wants up to T, at an hourly wage of \$$w$ per hour. The price of one unit of consumption is p.

a) Find mathematical expressions for their commodity demand, and labor supply as a function of the exogenous variables, V, T, and w, p, and the parameters of the model

b) What restrictions does this utility function impose on the consumer's labor supply function? Do these seem realistic.

3. Given the income distributions

(1,2,2,5,5,5,7,11,11,12,20,21,22,24)

(2,3,3,4,4,5,7,7,11,11,12,20,21,24)

and a poverty line of z = 6, calculate the Sen poverty measure. Explain the values obtained for the two distributions.

4.

a)Use the two income distributions in the problem above to evaluate the Foster-Greer-Thorbecke poverty measure for $\alpha = 2$.

b) Pool the distributions to evaluate the poverty measure for the total population. Show that the measure is a weighted sum of the measures of the individual distributions. Assume a single subgroup.

5.

There are two senior advisors to the government, A and B, both of whom agree that the poverty line is at $4,000 for a single person. However, they have different equivalence scales. Mr. A believes that the scale factor in determining equivalent income should be 0.25 for each additional family member. Mrs. B suggests that the scale factor should be 0.75.

a. Find the poverty line for families of two, three, and four under both values of the scale factors 0.25 and 0.75.

b. Explain how Mr. A and Mrs. B must have very different views about income sharing within a family to end up with such different answers.

c. Suppose that the government is committed to provide welfare eligibility to every family below the poverty line. If this government wishes to keep total spending to a minimum, which of the two views should it support?

6.

Show that the parameters μ and σ of the lognormal are the mean and standard deviation of the ln y. Hint: find the mean and variance of $z = \frac{lny-\mu}{\sigma}$ using the expectation operator and the variance operator.

Table 1. Female Full-Time Year-Round Workers, 1999 (in thousands)

Income Class	Number	Proportion	Class Midpoint
1-5,000	633	0.015660	2,500
5,000-10,000	1,391	0.034413	7,500
10,000-15,000	4,059	0.100418	12,500
15,000-25,000	11,049	0.273348	20,000
25,000-35,000	9,433	0.233369	30,000
35,000-50,000	7,656	0.189406	42,500
50,000-75,000	4,185	0.103535	62,500
75,000+	2,015	0.049850	
Total	40,421		
Mean = $33,303			

7.

Consider the information in the chart above for female workers.

a)Estimate μ_y and σ_y^2 for the table.

b)Estimate μ and σ for the lognormal distribution.

c)Estimate the probability that a randomly selected female worker will have income between $40,000 and $60,000.

d)Estimate the probability that a randomly selected male (from our class discussion) will have an annual income above $60,000. Compare this with the probability that a female full time worker will have an annual income above $60,000. Comment.

8.

Using the parameters of the lognormal distribution estimated for the sample above, compute the MAD, MSE, CHISQ, and K-S statistics.

9.
For the table listed above:
a)Compute the parameters of a log-logistic distribution with *k* estimated to be 0.3691.
b)Compute the MAD, MSE, CHISQ, and K-S statistics.
c)Compare the fit in b) with the log normal fit of the problem above.

10.
Show that the parameter m is the log of the median income for the log-logistic model.

11.
Repeat problem 8 for the Singh-Maddala.