# INCOME REDISTRIBUTION AND LOCAL TAX PROGRESSIVITY--A RECONSIDERATION

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ABSTRACT

This paper investigates and clarifies the relation between global tax progressivity as measured by Musgrave and Thin's "effective progression" and pointwise (local) progression as measured by the elasticity of after-tax income with respect to pre-tax income (residual progression). Liu's (1985) vigorous defense of the global measure of effective progression is shown to rest upon an erroneous proposition of Jakobsson (1976). A counterexample is presented demonstrating that the link between residual progression and Lorenz domination is much weaker than the work of Liu and Jakobsson suggests.

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## INTRODUCTION

In a recent article in this journal, Pak-wai Liu (1985) extends the work of Jakobsson (1976) and Musgrave and Thin (1948) to reconsider the concept of global tax progressivity. As Davies, St-Hilaire and Whalley (1984, p. 642) emphasize, Liu appears to provide a rigorous defense of the Musgrave-Thin measure. The purpose of this paper is to point out an error in Liu's work--an error ultimately traceable to Jakobsson--which, when corrected, demonstrates that the link between local and global progressivity is not as close as Liu implies. In fact, in light of this error, Liu's main result, that the Musgrave-Thin measure is perfectly compatible with local progressivity, is invalidated.

In characterizing a tax schedule as globally progressive, Liu (1985) sets forth two requirements: first, the tax schedule should be pointwise (locally) progressive everywhere and, second, the tax schedule should distribute income after tax more equally. Liu invokes a result of Jakobsson to argue that the two requirements are fulfilled by the Musgrave-Thin measure of effective progression and, thus, that his analysis (1985, 397) ". . . provides a theoretical underpinning and rehabilitates Musgrave-Thins' measure as an appropriate index of global progressivity." The crucial result of Jakobsson which Liu invokes to make this powerful assertion states that "a tax schedule is globally progressive if and only if its residual income progression is less than unity everywhere. . ." (1985, 396). If valid, this result would coalesce what, at one time, were thought to be competing, alternative measures of the degree of progression--one capturing local properties of a tax structure and the other the redistributive effects of a tax structure. In short, consistent local progressivity would yield the same conclusions as Lorenz dominance and vice-versa.

The importance of this should not be understated because, if valid, the result would provide a compelling basis for discriminating among the many alternative measures of tax progression which now exist. This is in fact the essential content of Liu's paper. Unfortunately, Liu's argument is not valid. As we demonstrate, one can have local regressivity at some income levels and yet still have global progressivity. The reason for this is that Liu's argument relies in part on the "only if" portion of Jakobsson's result which, as stated, is in error. Consequently, the coalescence of the redistributive and local criteria for progressivity is incomplete. Local progressivity for all income levels implies Lorenz dominance but Lorenz dominance does not necessarily imply local progressivity for all income levels.

In the next section, we state Jakobsson's results in full and explain the error in logic made in the proof of the "only if" portion of his proposition. A specific counterexample to the "only if" statement is presented showing conclusively that this portion of the proposition is wrong. Brief concluding remarks follow.

RESIDUAL PROGRESSION AND LORENZ-DOMINATION

Residual income progression, denoted a(y), is the elasticity of income after tax, x(y), with respect to income before tax, y. Thus a(y) takes on a value of one under a proportional tax, a value less than one for a progressive tax, and a value greater than one under a regressive tax; the smaller the value of $a(y)$, the more progressive the tax at income level y.

The Lorenz curve, L, for a given distribution of income, F, is implicitly defined by

$$L(F(\bar{x})) = \mu^{-1}\int_0^{\bar{x}} x \, dF(x), \tag{1}$$

where $\mu$ is the mean of the distribution of x. If F and G denote two distinct distributions of income, then using Jakobsson's terminology F is said to Lorenz dominate G (denoted F LD G) if and only if $L(G) \leq L(F)$ (i.e., $L(G(x)) \leq L(F(x))$ for all x). That is F LD G if and only if F generates a Lorenz curve that lies everywhere above the Lorenz curve generated by G.

We are now prepared to state Jakobsson's Proposition 1.

Proposition 1 (Jakobsson): Consider two tax systems with residual progressions given by $a_1(y)$ and $a_2(y)$. Let G denote the pre-tax distribution of income to which the tax systems are being applied, and let $F_1$ and $F_2$ denote the respective after-tax distributions of income.

(i) If $a_1(y) < a_2(y)$ for all y, then $F_1$ LD $F_2$.

(ii) If $F_1$ LD $F_2$ for any G, then $a_1(y) < a_2(y)$ for all y.

Liu's argument is based on a corollary which can be derived from Jakobsson's proposition as follows. Regard the pre-tax distribution of income as being generated by a proportional tax system with a zero tax rate for all y. Thus for this "tax", residual progression would be unity at all y. Since a tax system is progressive if and only if $a(y) < 1$, application of the proposition yields the following result:

Corollary (Jakobsson): $a(y) < 1$ for all y if and only if F LD G, where G and F are, respectively, the before-and-after-tax distributions of income.

We will now demonstrate that part (ii) of the proposition and the corresponding "only if" portion of the corollary are invalid. The following tax system applied to a four-person society refutes the corollary directly and thus demonstrates the falsity of the more general statement in part (ii) of the proposition.

| Person Number | y | Tax | x(y) | L(G(y)) | L(F(x)) | a(y) |
|---|---|---|---|---|---|---|
| 1 | 10 | 2 | 8 | .10 | .1067 | – |
| 2 | 20 | 3 | 17 | .30 | .3333 | 1.08 |
| 3 | 30 | 7 | 23 | .60 | .64 | 0.75 |
| 4 | 40 | 13 | 27 | 1.00 | 1.00 | 0.56 |
| | 100 | 25 | 75 | | | |

First note that the Lorenz curve of the post-tax distribution, F, lies everywhere above that of the pre-tax distribution G, i.e. F LD G. However, the residual progression coefficient in moving from an income level of \$10 to \$20 is found to be greater than one, contradicting the "only if" portion of the corollary.[1] Thus, part (ii) of the proposition is also rendered invalid.

Our counterexample clearly indicates that Jakobsson erred in his proof of part (ii) of the proposition. A careful reading reveals that Jakobsson has proven not part (ii) as stated, but rather the following much more restrictive statement:

(ii)* If $F_1$ LD $F_2$ for <u>all</u> G, then $a_1(y) < a_2(y)$ for all y.

Similarly, the "only if" portion of the corollary should read:

If a tax system is such that for all pre-tax distributions G, F LD G, then $a(y) < 1$ for all y.

This correction in the statement of the proposition and corollary explains how we were able to produce the above counterexample.[2] The implication of this correction is that link between residual proression and Lorenz domination is much weaker than indicated by Liu and Jakobsson. Thus, the attractiveness of the Musgrave-Thin measure is not as great as Liu asserts because his extension of Jacobsson is laid on faulty ground.

CONCLUSION

In a widely cited paper, Jakobsson states the result that a tax system is globally progressive in the sense of Lorenz dominance if and only if the tax structure is locally progressive everywhere. The importance of this result lies in the immediate implication (false as we show) that consensual judgements about a tax's redistributive impact can be made if and only if local progressivity prevails throughout the tax structure. Two extensively discussed characteristics of taxation, income redistribution and local tax progressivity, are thus apparently coalesced.

In a seemingly logical extension of Jakobsson's work, Pak-wai Liu employs the aforementioned result to provide a theoretical underpinning for the Musgrave-Thin measure of effective progression. Liu argues that a tax schedule should be classified as progressive, first, if it is locally progressive throughout and, second, if it redistributes income after tax more equally. Invoking Jakobsson, Liu concludes that the Gini-based Musgrave-Thin measure captures these two "natural" requirements for global progressivity.

In this paper, we demonstrate that Liu and Jakobsson's work is in error. We construct a simple example in which a tax system distributes income more equally after tax and thus must be judged as globally progressive by the Musgrave-Thin index but yet is pointwise regressive for some income levels. This clearly undermines Liu's effort to obtain a perfect coalescence of the concepts of local and global progressivity. Simply put, local progressivity over the entire tax schedule implies Lorenz dominance but Lorenz dominance does not necessarily imply local progressivity everywhere. Thus the question of the redistributive impact of a tax can only be partially linked to the question of the local progressivity of the tax. Although it would be clearly useful to do so, one cannot collapse both questions into one as Liu and Jakobsson attempt to do. This fact is especially important since in applied work tax systems with pointwise regressive and progressive sections are quite common.

FOOTNOTES

(1) The calculations of residual progression presented in the tabulation are arc elasticities.

(2) Hemming and Keen (1983) extend the corrected version of Jakobsson's work by deriving a condition under which a tax system will generate an after-tax distribution of income that will Lorenz dominate all pre-tax income distributions. However, Hemming and Keen do not mention the fact that they correct the original statement of Jakobsson's proposition and corollary. We beleive it is important to clarify Jakobsson's error so as to prevent continued misinterpretation by such otherwise careful scholars as Liu (1985) and Sandmo (1983, p. 315).

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