ALTERNATIVE SPECIFICATION AND ESTIMATION METHODS FOR DETERMINING RANDOM COEFFICIENT BETAS: COMPARISON AND EXTENSIONS

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I. Introduction

Recently Fabozzi and Francis (1978), Sunder (1980) and Lee and Chen (1980), used the random coefficient model developed by Hildreth and Houck (1968) and Theil (1971) to study the stochastic nature of the beta coefficients obtained from the market model. They have used various specifications of the market model and assumed arbitrarily that the covariance between the error term of regression and the error term of beta coefficient generating process is zero. In addition, Lee and Chen (1980) have found that multicollinearity problem exists in the estimation of the population variance of the stochastic beta coefficients.

The main purposes of this paper are (1) to analytically compare alternative models used to estimate random coefficient betas, (2) to investigate the covariance between the error term of regression and the error term of beta coefficient generating process, (3) to experiment with simplification schemes to alleviate multicollinearity problem, (4) to use specification tests to determine whether the OLS or the GLS estimation method should be used to estimate the random coefficient parameters, and (5) to discuss the possible new implications of random coefficent models to risk decomposition and portfolio diversification analyses. In the second section, basic models used by previous researchers are reviewed. A new model to generalize previous research is proposed. In the third section, a specification test derived by Ramsey and Schmidt (1976) for testing the appropriateness of new random coefficient model is defined. Potential implications of this method to alternative methods of estimating random coefficient parameters are discussed. In the fourth section empirical studies of using 451 firms are presented. Implications associated with these results in risk decomposition and risk diversification are discussed. Especially, the covariance risk is highlighted in the generalized risk decomposition model. Finally results of this study are summarized and the concluding remarks are indicated.

II. Alternative Random Coefficient Model Specifications

There exist two alternative fixed coefficient market models for estimating beta coefficients.

(1) $$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}, \quad t = 1, 2, \ldots, n$$

(2) $$R_{it} - R_{ft} = \alpha_i' + \beta_i'(R_{mt} - R_{ft}) + e_{it}'$$

where R_{it}, R_{mt}, and R_{ft} are rates of return for the i-th security, market rates of return and risk-free rates in the t-th period. Equation (1) is a non-risk premium type of market model [NRPMM] and equation (2) is a risk premium type of market model [RPMM] [see Fama (1976)]. Since Theil's (1971) procedure for estimating the variance of the random coefficient, ${}_i\sigma_1^2$, is derived under the assumption that the intercept of the regression is zero and equation (1) may be subject to specification error of omitting the R_{ft} as pointed out by Roll (1969), we will follow Lee and Chen (1980) to use equation (2) for estimating the random coefficient parameter. It is well-known that the intercept α_i' in equation (2) is theoretically equal to zero for the RPMM. Hence, equation (2) is a more appropriate model to be used in terms of Theil's method for estimating random coefficient beta. Let

$$Y_{it} = R_{it} - R_{ft} \text{ and } X_{it} = R_{mt} - R_{ft},$$

then equation (2) can be written as

(3) $Y_{it} = \beta_{it}X_t + e'_{it}$

$$\text{Where } E\begin{bmatrix} e_{it} \\ \\ \beta_{it} \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \beta'_{it} \end{bmatrix}, \ \text{Var}\begin{bmatrix} e_{it} \\ \\ \beta_{it} \end{bmatrix} = \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \\ \sigma_{01} & \sigma_1^2 \end{bmatrix}$$

$$E\begin{bmatrix} e_{it} \\ \\ \beta_{it} \end{bmatrix} [e_{it'}, \beta_{it'}] = 0 \text{ for } t \neq t'$$

Theil and Mennes (1959) derived a multiple regression for estimating σ_0^2, σ_1^2 and σ_{01} as

(4) $$\hat{e}'^2_t = \sigma_0^2 P_t + 2\sigma_{01} R_t + \sigma_1^2 Q_t + f_t$$

where

(5)
$$P_t = (1 - \frac{X_t^2}{\Sigma X_t^2}) \quad \text{(A)}$$
$$R_t = X_t(1 - 2\frac{X_t^2}{\Sigma X_t^2} + X_t \frac{\Sigma X_t^3}{(\Sigma X_t^2)^2}) \quad \text{(B)}$$
$$Q_t = X_t^2(1 - 2\frac{X_t^2}{\Sigma X_t^2} + \frac{\Sigma X_t^4}{(\Sigma X_t^2)^2}) \quad \text{(C)}$$

If $\sigma_{01} = 0$, then equation (4) reduces to

(6) $$\hat{e}_t^2 = \sigma_0^2 P_t + \sigma_1^2 Q_t + f_t$$

Equation (6) is the specification used by all previous empirical studies in investigating the randomness of beta coefficients. Fabozzi and Francis (1978), Alexander and Bensen (1982) and others have rewritten equation (1) as a deviation form for estimating the random coefficient beta. The specification used by these authors can be defined as

(7) $$r_{it} = \beta_{it} r_{mt} + e_{it}$$

where $r_{it} = R_{it} - \overline{R}_{it}$, $r_{mt} = R_{mt} - \overline{R}_{mt}$

Both r_{it} and r_{mt} are then used to estimate the random coefficient parameters in terms of Theil's (1971) specification. This approach might not be correct. The major restriction assumption of using this approach is the definition of intercept.[1] Sunder (1980) realizes this problem and derives a new multiple regression for estimating random coefficient parameter. The specification of market model used by Sunder is

(8) $$R_{it} = \alpha_i + \beta_{it} R_{mt} + e_{it}$$

In equation (8), intercept α_i is a fixed instead of random coefficient. Using equation (8) and procedure used by Theil (1971) Sunder obtained equation (9) for estimating the random coefficient parameters.

(9) $$\hat{e}^2_{it} = \sigma^2_0 A_{1t} + \sigma^2_1 A_{2t} + w_t$$

where

$$A_{1t} = 1 - \frac{1}{n} - \frac{(R_{mt} - \overline{R}_{mt})^2}{\Sigma(R_{mt} - \overline{R}_{mt})^2}$$

$$A_{2t} = R^2_{mt}\left(1 - \frac{2}{n} - \frac{2(R_{mt} - \overline{R}_{mt})^2}{\Sigma(R_{mt} - \overline{R}_{mt})^2}\right) + \frac{\Sigma R^2_{mt}}{n^2} + \frac{(R_{mt} - \overline{R}_{mt})^2}{[\Sigma(R_{mt} - \overline{R}_{mt})^2]^2}\Sigma R^2_{mt}(R_{mt} - \overline{R}_m)^2 + \frac{2(R_{mt} - \overline{R}_{mt})}{n\Sigma(R_{mt} - \overline{R}_{mt})^2}[\Sigma R^2_{mt}(R_{mt} - \overline{R}_{mt})]$$

If equation (1) is used to estimate the related parameters associated with random coefficient for market model, then equation (9) instead of equation (6) should be used. Hence, Fabozzi and Francis (1978) and Alexander and

Bensen (1982) has used a misspecified equation to estimate random coefficient parameters. Two major sources contribute the misspecification: (i) Theil's regression equation does not allow an intercept term. This mistake has been made by Fabozzi and Francis (1978) and Alexander and Benson (1982). (ii) The covariance term, σ_{01} is not necessarily negligible. The second potential mistake has not been recognized by any of the previous studies. In this study, we will test the importance of the coefficient associated with this covariance term.

III. Alternative Methods for Estimating Random Coefficients

There are three alternative methods which can be used to estimate the random coefficient parameters. These methods are (i) the Maximum Likelihood [ML] method used by Warren and Hildreth (1977), Bos (1982). (ii) The generalized least squares method [GLS] used by Theil (1971), Hildreth and Houck (1968), Fabozzi and Francis (1978), Alexander and Benson (1978). And (iii) the ordinary least squares (OLS) method used by Lee and Chen (1980) and Sunder (1980).

Theoretically, either the ML or the GLS method is a generalized method of the OLS method. However, empirically the OLS method might be more suitable than the GLS method in estimating the random coefficient parameters. The investigation of this issue can be regarded as an additional contribution of this study. Obenchain (1975) used the mean square error (MSE) criteria to derive the condition for the ordinary least squares (OLS) to be superior to both Gauss-Markov generalized least squares (GLS) and the biased least square (BLS) associated with ridge regression. He found that the OLS or the diagonally weighted least squares estimates are superior to both GLS and BLS when the problem is to estimate possible lack-of-fit (or misspecification) model.

To investigate whether equations (4) and (6) are misspecified or not, the technique of testing misspecification error developed by Ramsey and Schmidt (1976) will be used to do empirical study in the next section. Ramsey and Schmidt's procedure will be described as following.

Using the specification analysis technique, Ramsey and Schmidt derived the alternative approaches as defined in equation (10) for testing the misspecification of equation (4) or equation (5).[2]

$$\hat{f}_t = a_0 + a_1 q_{1t} + a_2 q_{2t} + a_3 q_{3t} + \tau_t \qquad (10)$$

where $\hat{f}_t = \hat{e}'^2_t - \hat{\hat{e}}'^2_t$, the OLS residuals associated with either equation (4) or equation (6).

$$q_{1t} = M[(\hat{\hat{e}}'^2_t)^2]$$

$$q_{2t} = M[(\hat{\hat{e}}'^2)^3]$$

$$q_{3t} = M[(\hat{\hat{e}}'^2)^4]$$

$$M = I - Z(Z'Z)^{-1}Z'$$

where Z is the data matrix associated with independent variables of either equation (4) or (6). The F value associated with (10) can be used to test whether the linear model as indicated in equation (9) is misspecified or not.

IV. Empirical Work

In this section data from 451 firms during 1965-1977 are used to investigate how alternative estimation methods and specifications can affect the random coefficient beta estimates. There are two major specifications as defined in equations (4) and (6) for testing degree of randomness associated with beta coefficient. Under each specification, both OLS and GLS can be used to estimate the random coefficient parameter.

(A) $\sigma_{01} = 0$ Case

If σ_{01} is assumed to be zero, then equation (6) is used to estimate the σ_1^2. Both OLS and GLS σ_1^2 estimates are calculated and the results are presented in Table 1. The OLS estimate of σ_1^2 is obtained by applying the OLS estimation procedure to equation (6). Theil (1971) has shown that the estimated residuals of equation (6) $\hat{f}_{it}$ is distributed as

(11) $$E(\hat{f}_{it}) = 0$$

$$\text{Var}(\hat{f}_{it}) = 2(\sigma_0^2 P_t + \sigma_1^2 Q_t)^2$$

Hence is the weighted least squares should be used to estimate σ_1^2. The weight to be used is $\sqrt{2}(\hat{\sigma}_0^2 P_t + \hat{\sigma}_1^2 Q_t)$. The rule used to determine whether the OLS estimate or the GLS estimate of σ_1^2 should be used has been discussed in the previous section.

TABLE 1: The Number of Companies with Significant $\hat{\sigma}_1^2$

Estimation Method	Level of Significance 5%	1%
OLS	231	216
GLS	72	37

Table 1 indicates that the estimated $\hat{\sigma}_1^2$'s obtained from the OLS are statistically much more significant than those obtained from the GLS. If the OLS method is used there are 231 and 216 estimated $\hat{\sigma}_1^2$ significantly different from zero at 5% and 1% significant level respectively. If the GLS is used, then these are only 72 and 37 estimated $\hat{\sigma}_1^2$ different from zero at 5% and 1% significant level respectively.

Now the possible problems associated with the procedure of using Theil's method in estimating σ_1^2 is discussed. One of the problems in applying Theil's procedure is the high multicollinearity between P_t and Q_t. In our sample, the simple correlation between P_t and Q_t is -0.9986. According to Theil and Mennes (1959) the problem of multicollinearity can be alleviated by simplifying P_t and Q_t as 1 and X_t^2. This is reasonable when sample size is large and X_t is small. In our study, the sample size is 156 and X_t is the difference between the market rate of return and the risk-free rate of return, which is a very small decimal. With the simplification, equation (6) becomes (12).

$$(12) \qquad \hat{e}'^2_{it} = \sigma_0^2 + \sigma_1^2 X_t^2 + f_t$$

The step of estimating GLS σ_0^2 and σ_1^2 is:
First, regressing $\hat{e}'^2_{it}$ on X_t^2 yields S_0^2 and S_1^2. Second, dividing $\hat{e}^2_{it}$, 1 and X_t^2 by $\sqrt{2}(S_0^2 + S_1^2 X_t^2)$ and then applying OLS yields $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$.

Again we can use the OLS estimate and the GLS estimate $\hat{\sigma}_1^2$ and S_1^2 to test whether a beta coefficient is random. The empirical results are shown in Table 2.

TABLE 2: The Number of Companies with Significant Random Beta Coefficients

Estimation Method	Level of Significance 5%	1%
OLS	256	222
GLS	66	33

By comparing the results of Table 2 with those of Table 1, it is found that the multicollinearity problem is alleviated by using the simplified specification as defined in equation (12). In other words, more OLS estimated σ_1^2's are significantly different from zero when equation (12) instead of equation (6) is used.

(B) $\sigma_{01} \neq 0$ Case

If the covariance between equation error term and the random errors of beta coefficient is not zero, then equation (4) should be used to test the randomness of beta coefficient. To alleviate the problem of multicollinearity, P_t, R_t and Q_t of equation (4) are replaced by 1, x_t and x_t^2. The weight used in the GLS estimation is $\sqrt{2}[\hat{\sigma}_0^2 + (2\hat{\sigma}_{01}) x_t + \hat{\sigma}_1^2 x_t^2]$. The empirical results associated with equation (4) are reported in Tables 3 and 4 respectively. Table 3 presents the number of companies with significant σ_1^2 estimates and Table 4 presents the number of companies with significant σ_{01}.[3] There are 131 and 175 OLS σ_{01} estimates which are significantly different from zero under 1% and 5% significance levels. This implies that equation (6) might be a misspecified equation for testing the randomness of beta coefficients.

TABLE 3: The Number of Companies with Signficant Random Beta Coefficients

Estimation Method	Significance Level 1%	5%
OLS	206	244
GLS	41	67

TABLE 4: The Number of Companies with Significant Covariance Term

Estimation Method	Significance Level 1%	5%
OLS	131	175
GLS	40	74

To detect the possible misspecification associated with equations (6) and (4) the Ramsey and Schmidt (1976) specification error test as defined in equation (10) is used to do the analysis. It is found that there are about 40% of firms with specification errors under 5% significant level for both specifications (6) and (4). Two possible implications associated with these results. First, these results imply that none of the previous researches has used a correct specification to investigate the possible randomness of beta and risk decomposition. Secondly, this misspecification results imply that Obenchain's (1973) results should be used to determine whether the OLS estimates or the GLS estimates should be used to test the existence of random beta coefficients.

Based upon Obenchain's (1975) MSE criteria of selecting alternative estimation methods as discussed in Section III.OLS estimates of σ_1^2 and σ_{01} might give us more information than those of GLS. Most recently Bos (1982) used the maximum likelihood (ML) method to test the randomness of beta coefficients. He found the OLS results are very similar to the ML results.[4] This finding further supports our argument that the OLS instead of the GLS should be used to estimate the random coefficient parameter.

Following Lee and Chen (1980) and Lee and Chen (1982), random coefficient results imply that the standard risk decomposition method should be modified. Use Lee and Chen's approach, the implications of our new findings associated with covariance term on risk decomposition process are now explored. In accordance with equations (3), (4) and (5), it can be shown that

(12) $$\mathrm{Var}(Y_{it}) = \beta_i^2 \mathrm{Var}(X_{it}) + \sigma_0^2 + \sigma_{01}X_{it} + \sigma_1^2 X_{it}^2$$

where $\mathrm{Var}(Y_{it})$ = total risk

$\beta_i^2 \mathrm{Var}(X_{it})$ = systematic risk

σ_0^2 = pure residual risk

$\sigma_1^2 X_{it}^2$ = beta estimation risk

$\sigma_{01}X_{it}$ = covariance risk

Under the fixed coefficient market model, we generally define the summation of σ_0^2, $\sigma_1^2 X_{it}^2$ and $\sigma_{01}X_{it}$ as nonsystematic risk. Under this generalized specification, we decompose the traditional nonsystematic into three components. The first component represents the pure nonsystematic risk; the second component represents the degree of randomness associated with beta generation process; the third component represents the comovement between the beta fluctuation and the fluctuation of pure residual term. Sunder (1980) found that the degree of randomness for portfolio betas is generally similar to those of individual security. This imply that $\sigma_1^2 X_{it}^2$ is generally not a diversifiable risk. $\sigma_{01}X_{it}$ represents comovement of two random terms and therefore, it is not necessarily a diversifiable risk which can be diversified by portfolio formulation. In sum, this generalized risk decomposition model will provide security analysts and portfolio managers with more information about the total risk components of both individual securities and portfolios. It might be fruitful to generalize Sharpe's (1963) diagonal model by using the generalized random coefficient model developed in this study.

V. Summary

In this paper impacts of alternative specification and estimation methods on random coefficient betas determination are theoretically and empirically investigated in some detail. It is found that the traditional specification model is misspecified. A better model is theoretically and empirically justified and proposed. It is also found that the OLS method is generally better than the GLS method for estimating random coefficient parameter estimate of beta coefficient. The comparisons of OLS estimate, GLS estimate and Maximum Likelihood estimate are to be done in the future research.

FOOTNOTES

[1] See Appendix A for the derivation.

[2] Cheng and Lee (1982) have used this kind of misspecification error test technique to identify the misspecification of alternative market models.

[3] In both tables, results from both OLS and GLS are presented. The justifications and interpretations of these results are similar to those of Case (A) in this section.

[4] Bos (1982) has also generalized the specification of beta coefficient to allow the autocorrelation of beta coefficient over time. However, he has found that the specification used in this paper is generally acceptable. It should be noted that the major weakness of Bos study is that he has not considered the covariance term.

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APPENDIX A

The relationship between the fixed coefficient intercept and the random coefficient intercept.

The fixed coefficient and the random coefficient market models can be defined as

(A-1) $R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$

(A-2) $R_{it} = \alpha_{it} + \beta_{it} R_{mt} + u_{it}$

The random coefficient market model of equation (A-2) can be written as

(A-3) $R_{it} = \alpha_i + \beta_i R_{mt} + u^*_{it}$

where $u^*_{it} = (\alpha_{it} - \alpha_i) + (\beta_{it} - \beta_i)R_{mt} + u_{it}$

If $\alpha_i = \overline{R}_{it} - \beta_i \overline{R}_{mt}$, then u^*_{it} can be rewritten

(A-4) $u^*_{it} = \alpha_{it} - \overline{R}_{it} + \beta_i \overline{R}_{mt} + (\beta_{it} - \beta_i)R_{mt} + u_{it}$

If $\alpha_{it} - \overline{R}_{it} = -\beta_{it}\overline{R}_{mt}$, then Equation (A-4) reduce to

(A-5) $u^*_i = (\beta_{it} - \beta_i)(R_{mt} - \overline{R}_{mt}) + u_{it}$

If (A-5) holds, then the model as indicated in equation (7) can be used to estimate the σ_1^2. The key issue is whether

$$\alpha_{it} - \overline{R}_{it} = -\beta_{it}\overline{R}_{mt}$$

holds or not. It follows from (A-2) that

$$\frac{1}{n}\sum_{t=1}^{n} R_{jt} = \frac{1}{n}\sum_{t=1}^{n} \alpha_{it} + \frac{1}{n}\sum_{t=1}^{n} \beta_{it}R_{mt} + \frac{1}{n}\sum_{t=1}^{n} u_{it}$$

$$\overline{R}_{it} = \overline{\alpha}_{it} + \frac{\frac{1}{n}\sum_{t=1}^{n} \beta_{it}R_{mt}}{\overline{R}_{mt}} \overline{R}_{mt} + \overline{u}_{it}$$

Consequently

$$\alpha_{it} - \overline{R}_{jt} = \alpha_{it} - \overline{\alpha}_{it} - \frac{\frac{1}{n}\sum_{t=1}^{n} \beta_{it}R_{mt}}{\overline{R}_{mt}} \overline{R}_{mt} - \overline{u}_{it} \neq -\beta_{it}\overline{R}_{mt}$$

Hence Equation (7) cannot be applied directly to estimate σ_1^2.