THE DEFINITION OF THE INDIVIDUAL IN LABOR SUPPLY ANALYSIS

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I. Introduction

It is commonplace to recognize that both commodity consumption and labor supply decisions take place in a family context. In the analysis of commodity demand the decision making process within the family need not be examined closely. In empirical studies of commodity demand, when a utility function is specified, the utility function is typically said to represent the family's or household's preferences. Hence,

> It (economic theory) assumes that each household makes consistent decisions, as though it were a single individual. Thus economists ignore many interesting problems of how the household makes its decisions. (Lipsey & Steiner, 1981, p. 45)

This is a cost minimizing approach to the analysis of commodity demand. Available data are measured at the level of the household, or a more aggregate level. This makes it difficult or impossible to disaggregate to the level of the family member. In addition, the benefits of such disaggregation are very low, given the interest of most economists in market level aggregates.[1]

However, in the analysis of labor supply behavior explicit recognition of the family context of decision making is critical. In analyzing labor supply behavior we must take account of cross-wage effects, that is, the effect of changes in one family member's wages on the labor supply of other family members. Analogous cross-price effects are absent in commodity choice decisions since all family members face the same commodity prices.

In the pure theory the individual is a primitive term having the property that it is the entity which has preferences or a utility function. Thus, it seems we have a choice between assuming that the utility function represents the preferences of the entire family, or assuming that each family member has their own preferences, each represented by a distinct utility function. These approaches may be characterized in terms of the definition of the individual used; the family as individual, or the family member as individual.

While both approaches have been used in empirical labor supply research the first approach is much more popular. In Section 2 we show that, even using the second approach and defining a separate utility function for each family member, a family utility function must be specified. In Section 3 we examine some alternative formulations of the family utility function under the second approach.

II. Two Definitions of the Individual

In this section we compare the two definitions of the individual discussed in the introduction. The first approach is to define a single utility function for the whole family.[2] The second approach is to define a separate utility function for each family member.[3] We show that, in view of the family context of labor supply decisions, it is impossible to avoid specifying a utility function for the whole family. In the interests of clarity and brevity, we restrict the analysis to the situation where there are only two family members with only a single consumption good to be traded off against leisure.[4]

When labor suply decisions are considered in a family context, it is reasonable to assume that family members pool their income. Thus, the family's income constraint is

$$C = w_1H_1 + w_2H_2 + y \qquad (1)$$

where C is total family consumption, w_i and H_i are the real wage rate and quantity of labor supplied by the i^{th} family member, and y is real non-wage income. Each family member is also faced with a time constraint

$$t = H_i + R_i \qquad (2)$$

where t is total time available and R_i is the leisure of the i^{th} family member. These constraints may be combined to form the family's full income constraint

$$F = C + w_1R_1 + w_2R_2 \qquad (3)$$

where $F = (w_1 + w_2)t + y$ is full income. In either approach, the individual (family or family member) is assumed to maximize utility subject to this constraint.

If we take the first approach, and interpret the individual to mean the family, then we must specify a family utility function. It is typically assumed that the family's utility depends on total family consumption, and on the leisure of the family members:

$$U = U(C,R_1,R_2) \qquad (4)$$

The utility function is assumed to be monotonically increasing, strictly quasi-concave, and twice continuously differentiable.

Given these regularity conditions there is a unique global maximum of U, subject to the constraint. The demand functions for the consumption good and for leisure will have the usual neoclassical properties. In particular, these demand functions will be single valued.[5] The property of single-valuedness is important since multiple or set-valued functions are impossible to estimate. Since this model, equations (3) and (4) is familiar, we will not analyze it further at this point.

If we want to interpret the family to mean a family member then we must specify a utility function for each family member. We will assume that the utility of each family member depends on total family consumption and on that person's leisure. Hence,

$$U^i = U^i(C,R_i) \qquad i=1,2 \qquad (5)$$

Each family member's utility function is assumed to satisfy the regularity conditions imposed on equation (4). We retain the assumption that family income is pooled, so the constraint is still given by (3). This is the approach taken by Leuthold (1968) and by Manser and Brown (1979, 1980).[6]

Each family member is assumed to maximize their own utility, U^i, subject to the constraint (3). Since the leisure taken by the other family members enters the constraint, the decisions of the family members are interdependent. One way to recognize this formally is to define the vector-valued function

$$V(C,R_1R_2) = (U^1(C,R_1),U^2(C,R_2))' \qquad (6)$$

Assuming each family member maximized their own utility is equivalent to assuming the family maximizes $V(\cdot)$.

For notational convenience, we let $Z = (C,R_1,R_2)$. We will say that Z^* is a global vector maximum if both (a) the constraint is satisfied at Z^* and (b) $V(Z^*) = V(Z)$ for all Z in the domain of V which satisfy the constraint. We will also use the following theorm:

> If there exists both (a) a vector of positive weights, denoted k, and (b) a Z^* which maximizes k'V subject to the constraint then Z^* is a vector maximum. (Takayama, 1974, p. 116)

Intuitively, if the weights, k, exist, then we can convert the problem of maximizing the vector-valued function V(Z) into the simpler problem of maximizing the scalar-valued function $T(Z,k) = k'V(Z)$.[7] We write T(Z,k) to emphasize the role of k. The converse of the theorem also holds, except that some of the weights may be zero. That is, if Z^* is a vector maximum, then there exists a non-negative vector of weights, k, such that Z^* is a global maximum of k'V.

The function T(Z,k) is a non-negative combination of strictly quasi-concave functions, hence T is strictly quasi-concave. Then for a given vector of weights, k, we can find the point Z^* which maximizes T(Z,k), subject to the constraint (3). This Z^* will be a unique global maximum of T. In general, this maximum point, Z^*, will depend on the vector of weights chosen, as well as on wages and non-wage income. Thus, we have $Z^* = Z^*(W,k)$ where $W = (w_1,w_2,y)$. Since Z^* satisfies the conditions of the theorem, Z^* is a vector maximum.

Now, while Z^* is a unique maximum of T, it is not necessarily a unique vector maximum. To see this, simply note that Z^* depends on the weights used to define T(Z,k). Allowing the weights to vary gives the set of vector maxima for given wage rates and non-wage income. We denote this set $Z^*(W)$. This is the set of Pareto-optimal combinations of consumption and family members' leisure.

The non-uniqueness of the vector maxima implies that the demands for consumption and family members' leisure will be set-valued when considered as functions of the wage rates and non-wage income only, where these set-valued demands are given by $Z^*(W)$. In order to obtain single-valued demands a vector of weights, k, must be given. The single-valued demands are then give by $Z^*(W,k)$ for the fixed value of k. The value of k determines which of the Pareto-optimal combinations in $Z^*(W)$ is actually chosen by the family.

If we interpret the individual to mean the family members, we obtain demand functions that are set-valued unless we specify the weights to be given the preferences of the various family members. At given wage rates and non-wage income only one point in $Z^*(W)$ will actually be chosen, say Z_o^*. Since only one point is chosen, we know that some weighting scheme, $k(Z_o^*)$, exists which rationalizes the choice. But the existence of these weights and the existence of a function, $T = k'V$, maximized by Z_o^* is the existence of a family utility function.

Thus, if we consider labor supply decisions in a family context, we are constrained to specify some form of family utility function if we wish to obtain estimable labor supply functions. The family utility function may be specified directly, as it is when the individual is taken to mean the family, or indirectly, as the weighted sum of family members' preferences. The family utility function may be specified implicitly by specifying family members' utility functions and a weight generating process. In the next section we consider alternative formulations of the family utility function resulting from different weight generating processes.

III. Family Utility Functions

In the preceding section we showed that when each family member maximizes own utility, and a particular combination of consumption and leisure is chosen by the family, then a family utility function exists. This family utility function is a weighted sum of the family members' utility functions. We can view the weights as determining the combination, Z_0^*, actually chosen. It seems reasonable to assume that some process exists which generates the weights, and thus determines the combination of consumption and family members' leisure actually chosen.[8] In this section we discuss the implications for observable labor supply behavior of three possible "weight-generating" processes; constant weights, expectations, and bargaining.

A. Constant Weights. We begin by analyzing the case where the weights to be given each family members' utility are constants. The primary appeal of this formulation is its simplicity. As before, we assume there are only two family members with only a single consumption good to be traded off against leisure. We assume that each family members' utility function is given by (5). Since the weights are non-negative, with at least one positive weight, the weights can be normalized to sum to one. Thus, we can write the family utility function as

$$T(C,R_1,R_2) = kU^1(C,R_1) + (1-k)U^2(C,R_2Y) \quad 0<k<1 \qquad (7)$$

Since the weight, k, is constant it can be treated as a parameter of the family utility function, T. Thus, the family utility function in (7) is simply a special case of the standard family utility function in (4). The assumption of constant weights is equivalent to assuming $\partial^2 U/\partial R_1 \partial R_2 = 0$ in (4). Thus, if we assume the weights are constant, the family member approach cannot be distinguished from the usual approach.

B. Expectations. An alternative approach is to assume that the weights are generated implicitly by family members' expectations about each others behavior. That is, we assume each family member choses consumption and own leisure to maximize own utility, given what that person expects the other family member to do.

Formally, each individual is assumed to maximize U^1 subject to the constraint (3), and to $R_j = e_i(R_j)$, where $e_i(R_j)$ denotes i's expectation of the amount of leisure taken by j. This implies that, for given wage rates and non-wage income, the optimal amounts of total family consumption and own leisure are, from 1's point of view, $C_1^*(W,e_1(R_2))$ and $R_1^*(W,e_1(R_2))$. Similarly, from 2's point of view, the optimal quantities are $C_2^*(W,e_2(R_1))$ and $R_2^*(W,e_2(R_1))$, where $W = (w_1,w_2,y)$.

In this approach we must distinguish between expected, actual, and optimal quantities, since these need not always be equal. To see this, consider the case where family members anticipate each others' behavior incorrectly, and, as a consequence, differ as to the optimal amount of family consumption. In this case, actual consumption need not be equal to either family members' desired consumption. Consequently, this approach includes a large number of models that may distinguished by the assumptions made about (1) the structure of the utility function, (2) the relation between actual and optimal quantities, and (3) how expectations are formed. The last two assumptions are important for the analysis of disequilibrium behavior.[9]

In equilibrium the actual, expected, and optimal quantities of consumption and leisure will be equal. The equilibrium quantities of leisure can be found by solving the simultaneous equations

$$R_1 = R_1(W,R_2) \tag{8}$$

$$R_2 = R_2(W,R_1)$$

The equilibrium quantity of consumption can then be found from the constraint.

Two related problems arise in the expectations approach. First, the implications for equilibrium behavior of the expectations approach may not be different from the implications of the neo-classical approach. For example, the Mincer (1962) - Kosters (1966) family labor supply model leads to the system of equations in (8). For another example, if both family members' utility functions are Stone-Geary, or if the usual family utility function is Stone-Geary, then we obtain the same labor supply functions.

This suggests that the expectations approach may be distinguishable from the usual approach only in disequilibrium, which leads to the second problem. Within the family, the costs of information about others' behavior is likely to be low, and adjustment to equilibrium rapid. If data is collected only infrequently, then observed behavior will be equilibrium behavior, and the parameters of the adjustment process may not be identifiable. This may be true even if the expectations model can be distinguished from the neo-classical model. Even if data on disequilibrium behavior are available, the econometric results are likely to be sensitive to the assumptions made about the adjustment of expectations and the adjustment of actual to optimal quantities.

C. Bargaining. A third approach is to assume that labor supply behavior is the outcome of an intra-family bargaining process. This approach has been explored by Manser and Brown (1979, 1980).

In their analysis Manser and Brown assume that both individuals have separate utility functions, $U^i(C,R_i)$.[10] Family income is assumed to be pooled, so that the constraint is given by equation (3). However, Manser and Brown distinguish between family members' and the households' non-wage income, that is, $y = y_1 + y_2 + y_h$, where y_i is i's non-wage income, and y_h is the none-wage income gained or lost as the result of the decision to form a household. Nonetheless, the bargaining approach clearly fits in the framework of Section II.

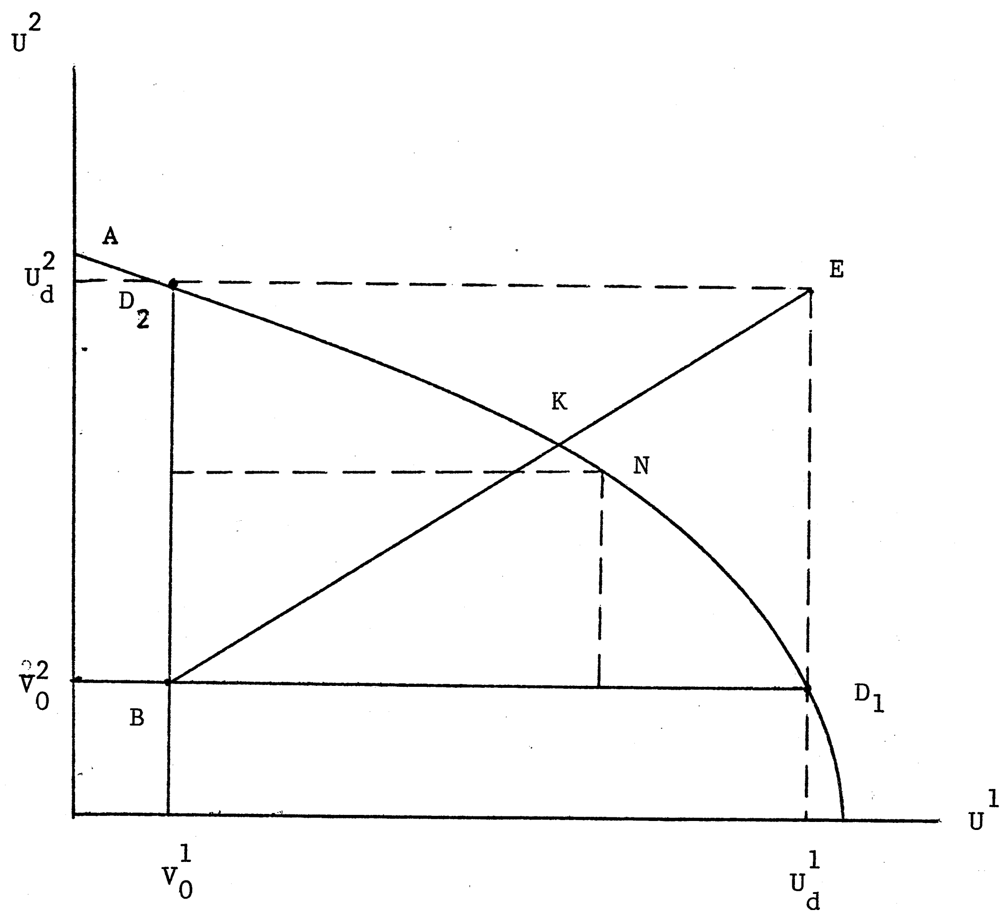
An important variable in bargaining analysis is the conflict point, or non-cooperative solution. In the context of family labor supply the conflict point is the maximum utility the individual would receive if single. It is convenient to write this in terms of the indirect utility function, hence, we let $V_0^i = V^i(w_i,y_i)$ denote the maximum utility i receives if single. Note that V_0^i depends only on i's wage and non-wage income, but not on j's. The conflict point must lie inside the utility possibility frontier (UPF) for the family, otherwise there are no gains to household formation.

Manser and Brown analyze three bargaining solutions, the dictatorial, Nash, and Kalai-Smorodinski (1975) solutions. Under all three bargaining rules the outcome is unique, Pareto optimal, and invariant to linear transformations of the utility functions. The Nash and Kalai-Smorodinski (K-S) solutions are symmetric. The Nash solution has the property of independence of irrelevant alternatives. The K-S solution replaces this property with a monotonicity property. The K-S monotonicity property requires that, for any given level of i's utility, if the situation changes so that j's utility can be increased, then the solution must increase j's utility.

The outcomes of the bargaining process under the alternative solutions are represented graphically in Figure 1, which is drawn for fixed values of the wage rates and non-wage incomes. The curve AA in Figure 1 is the family's UPF, point B is the conflict point. Under the dictatorial solution the dictator captures all of the gains. The outcome is at D_1 or D_2 according as 1 or 2 is the dictator. The K-S solution can be found by drawing the line from the conflict point to point $E = (U_d^1, U_d^2)$. The K-S solution is the intersection of this line with the UPF, point K in Figure 1. The Nash solution maximizes the geometric average of the gains from bargaining; this is shown as point N.

All three solution concepts yield unique outcomes. Since each point on the UPF corresponds to a different Pareto optimum, and thus to a unique combination of consumption and leisure, (C,R_1,R_2), the demand functions are single-valued. In general, each bargaining rule yields a different outcome, and, as a result, implies different demand functions. However, if the bargaining situation is symmetric the Nash and K-S solutions coincide (Roth, 1979, p. 102).[11] We cannot

Figure 1 Bargaining Outcomes



distinguish further between the demand functions implied by these bargaining solutions at this level of generality.

The demand functions derived from the bargaining model can be distinguished empirically from the neoclassical demand functions. First, the bargaining models all imply that the substitution matrix is not symmetric. Second, the bargaining models also imply that the y_i and y_h enter the demand functions independently, rather than as the summary.[12] Manser and Brown (1979) report that the neoclassical restrictions of symmetry and equal income effects are rejected.[13]

IV. Summary

In analyzing labor supply behavior it is important to recognize the family context of decision making. A necessary condition for the empirical interpretation of a model of family labor supply is the single-valuedness of the demand functions for the consumption goods and leisure. The usual approach has been to specify a family utility functions which represents the preferences of all family members. An alternative approach is to assume each family member has a separate utility function. These approaches may be characterized by the definition of the individual used. We show that, in the separate utility functions approach, the requirement of single-valued demands implies the existence of a family utility function. This family utility function is a weighted average of family members' utility functions.

We also show that the weighted average utility function always exists, ex post. If the family chooses a unique combination (C, R_1, R_2)

at given wage rates and non-wage income, then there exists a set of weights such that the weighted average of family members' utilities is maximized by the combination chosen. However, since the weights depend on the point chosen, the ex post weighted average simply says whatever the family does is optimal.

We can also view the weights as being generated by some process, ex ante. In Section III we discussed three alternative "weight generating" processes, constant weights, expectations, and bargaining. The assumption that weights are constant yields a special case of the usual family utility function. If we assume that the weights are generated by family members' expectations about each others' behavior then the model is not always distinguishable from the usual approach. While the expectations approach may be useful in modeling individual adjustment behavior, it may be more fruitful to begin analyzing disequilibrium behavior in the standard framework. The third alternative examined is the assumption that the weights are generated by a bargaining process. The bargaining models imply that the substitution matrix is nonsymmetric and that property rights in non-wage income influence labor supply behavior. The bargaining approach needs to be examined further, both theoretically and empirically.

NOTES

1. This is not to say that the composition of the household can be neglected. Demographic variables should be included to (partially) control for variations in preference.

2. This is the standard labor supply model. For example, all of the studies of family laobr supply reviewed by Keeley (1981) use this approach.

3. The interconnected utility functions approach (e.g. Becker, 1974) can be viewed as imposing a separability restriction on the family utility function.

4 The argument presented here can readily be generalized to situations with more family members and more commodities. We use total family consumption, rather then the consumption of each family member, since we are primarily interested in labor supply.

5. A function is single-valued if only one point in the range of the function can be associated with each point in the domain. If more than one point in the range can be associated with a point in the domain the function is set or multi-valued, or a correspondence.

6. Much of Manser and Brown's analysis deals with the decision to form a household. We are assuming that the household is, and will remain, formed.

7. Assuming that the family maximizes V is analogous to assuming that a multi-product firm maximizes output. The weights in T are analogous to the prices in the firms' profit function.

8. At least conceptually. Some functional form other than $T = k(P)'V(Z)$ (where P is the weight generating process) may be more convenient analytically.

9. In particular, for the analysis of the adjustment behavior of individuals to changes in wage rates and non-wage income.

10. The utility functions are assumed to be von Neuman-Morgenstern. In their later paper, Manser and Brown extend their analysis to include multiple consumption goods. In multiple good situations, preferences toward risk and toward goods are confounded (Kihlstrom and Mirman, 1974).

11. If the bargaining situation is symmetric there is only one Pareto optimum. Then any solution requiring symmetry and Pareto optimality will coincide with the Nash solution (Roth, 1979, pp. 6-12).

12. These results require that the conflict point depend on prices.

13. The empirical results are conditional on the Rotterdam model used by Manser and Brown, and on the data (NLS survey data). Also, see the criticisms of Ashenfelter (1979).

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